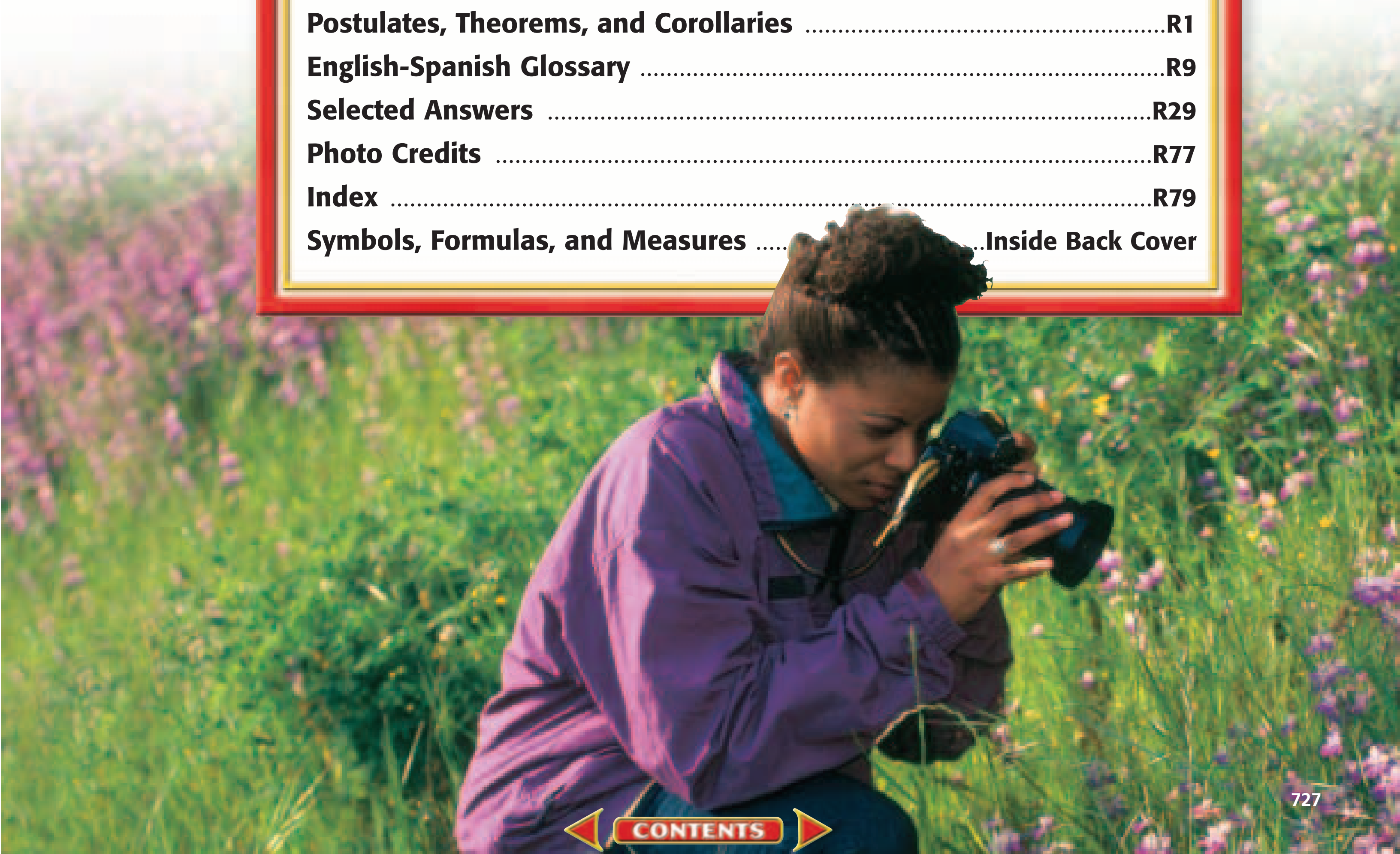
Student Handbook

Skills

Reference

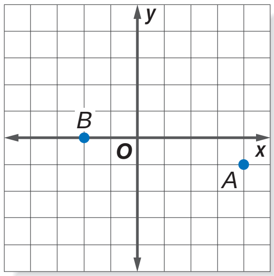


Prerequisite Skills

Graphing Ordered Pairs

- Points in the coordinate plane are named by **ordered pairs** of the form (x, y). The first number, or **x-coordinate**, corresponds to a number on the x-axis. The second number, or **y-coordinate**, corresponds to a number on the y-axis.

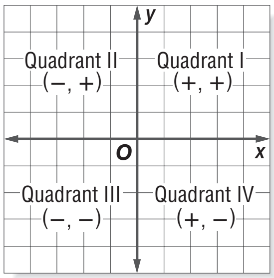
Example 1 **Write the ordered pair for each point.**



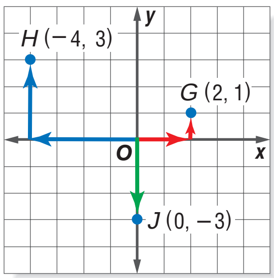
a. *A*
The x-coordinate is 4.
The y-coordinate is -1.
The ordered pair is $(4, -1)$.

b. *B*
The x-coordinate is -2.
The point lies on the x-axis, so its y-coordinate is 0.
The ordered pair is $(-2, 0)$.

- The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axes intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



Example 2 **Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**



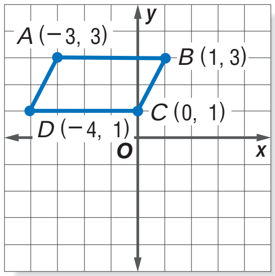
a. ***G*(2, 1)**
Start at the origin. Move 2 units right, since the x-coordinate is 2. Then move 1 unit up, since the y-coordinate is 1. Draw a dot, and label it *G*. Point *G*(2, 1) is in Quadrant I.

b. ***H*(−4, 3)**
Start at the origin. Move 4 units left, since the x-coordinate is -4. Then move 3 units up, since the y-coordinate is 3. Draw a dot, and label it *H*. Point $H(-4, 3)$ is in Quadrant II.

c. ***J*(0, −3)**
Start at the origin. Since the x-coordinate is 0, the point lies on the y-axis. Move 3 units down, since the y-coordinate is -3. Draw a dot, and label it *J*. Because it is on one of the axes, point $J(0, -3)$ is not in any quadrant.

Example 3 **Graph a polygon with vertices $A(-3, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-4, 1)$.**

Graph the ordered pairs on a coordinate plane.
Connect each pair of consecutive points.
The polygon is a parallelogram.

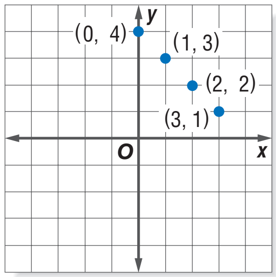


Example 4 **Graph four points that satisfy the equation $y = 4 - x$.**

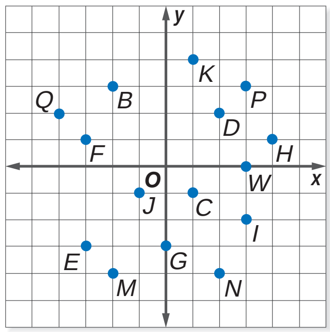
Make a table.
Choose four values for x.
Evaluate each value of x for $4 - x$.

x	4 − x	y	(x, y)
0	4 − 0	4	(0, 4)
1	4 − 1	3	(1, 3)
2	4 − 2	2	(2, 2)
3	4 − 3	1	(3, 1)

Plot the points.



Exercises **Write the ordered pair for each point shown at the right.**



1. *B* **2.** *C* **3.** *D*
4. *E* **5.** *F* **6.** *G*
7. *H* **8.** *I* **9.** *J*
10. *K* **11.** *W* **12.** *M*
13. *N* **14.** *P* **15.** *Q*

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

16. $M(-1, 3)$ **17.** $S(2, 0)$ **18.** $R(-3, -2)$ **19.** $P(1, -4)$
20. $B(5, -1)$ **21.** $D(3, 4)$ **22.** $T(2, 5)$ **23.** $L(-4, -3)$
24. $A(-2, 2)$ **25.** $N(4, 1)$ **26.** $H(-3, -1)$ **27.** $F(0, -2)$
28. $C(-3, 1)$ **29.** $E(1, 3)$ **30.** $G(3, 2)$ **31.** $I(3, -2)$

Graph the following geometric figures.

32. a square with vertices $W(-3, 3)$, $X(-3, -1)$, $Y(1, 3)$, and $Z(1, -1)$
33. a polygon with vertices $J(4, 2)$, $K(1, -1)$, $L(-2, 2)$, and $M(1, 5)$
34. a triangle with vertices $F(2, 4)$, $G(-3, 2)$, and $H(-1, -3)$
35. a rectangle with vertices $P(-2, -1)$, $Q(4, -1)$, $R(-2, 1)$, and $S(4, 1)$

Graph four points that satisfy each equation.

36. $y = 2x$ **37.** $y = 1 + x$ **38.** $y = 3x - 1$ **39.** $y = 2 - x$

Changing Units of Measure within Systems

Metric Units of Length
1 kilometer (km) = 1000 meters (m)
1 m = 100 centimeters (cm)
1 cm = 10 millimeters (mm)

Customary Units of Length
1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 ft
1 mile (mi) = 5280 ft

- To convert from larger units to smaller units, multiply.
- To convert from smaller units to larger units, divide.

Example 1 **State which metric unit you would use to measure the length of your pen.**

Since a pen has a small length, the *centimeter* is the appropriate unit of measure.

Example 2 **Complete each sentence.**

a. 4.2 km = __?__ m

There are 1000 meters in a kilometer.

4.2 km × 1000 = 4200 m

b. 125 mm = __?__ cm

There are 10 millimeters in a centimeter.

125 mm ÷ 10 = 12.5 cm

c. 16 ft = __?__ in.

There are 12 inches in a foot.

16 ft × 12 = 192 in.

d. 39 ft = __?__ yd

There are 3 feet in a yard.

39 ft ÷ 3 = 13 yd

- Example 3 involves two-step conversions.

Example 3 **Complete each sentence.**

a. 17 mm = __?__ m

There are 100 centimeters in a meter. First change *millimeters* to *centimeters*.

17 mm = __?__ cm — smaller unit → larger unit

17 mm ÷ 10 = 1.7 cm — Since 10 mm = 1 cm, divide by 10.

Then change *centimeters* to *meters*.

1.7 cm = __?__ m — smaller unit → larger unit

1.7 cm ÷ 100 = 0.017 m — Since 100 cm = 1 m, divide by 100.

b. 6600 yd = __?__ mi

There are 5280 feet in one mile. First change *yards* to *feet*.

6600 yd = __?__ ft — larger unit → smaller unit

6600 yd × 3 = 19,800 ft — Since 3 ft = 1 yd, multiply by 3.

Then change *feet* to *miles*.

19,800 ft = __?__ mi — smaller unit → larger unit

19,800 ft ÷ 5280 = $3\frac{3}{4}$ or 3.75 mi — Since 5280 ft = 1 mi, divide by 5280.

Metric Units of Capacity
1 liter (L) = 1000 millimeters (mL)

Customary Units of Capacity	
1 cup (c) = 8 fluid ounces (fl oz)	1 quart (qt) = 2 pt
1 pint (pt) = 2 c	1 gallon (gal) = 4 qt

Example 4 **Complete each sentence.**

a. 3.7 L = __?__ mL

There are 1000 milliliters in a liter.

3.7 L × 1000 = 3700 mL

b. 16 qt = __?__ gal

There are 4 quarts in a gallon.

16 qt ÷ 4 = 4 gal

• Examples c and d involve two-step conversions.

c. 7 pt = ___?___ fl oz

There are 8 fluid ounces in a cup.
First change *pints* to *cups*.

7 pt = ___?___ c

7 pt × 2 = 14 c

Then change *cups* to *fluid ounces*.

14 c = ___?___ fl oz

14 c × 8 = 112 fl oz

d. 4 gal = ___?___ pt

There are 4 quarts in a gallon.
First change *gallons* to *quarts*.

4 gal = ___?___ qt

4 gal × 4 = 16 qt

Then change *quarts* to *pints*.

16 qt = ___?___ pt

16 qt × 2 = 32 pt

• The mass of an object is the amount of matter that it contains.

Metric Units of Mass
1 kilogram (kg) = 1000 grams (g)
1 g = 1000 milligrams (mg)

Customary Units of Weight
1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2000 lb

Example 5 **Complete each sentence.**

a. 2300 mg = ___?___ g

There are 1000 milligrams in a gram.

2300 mg ÷ 1000 = 2.3 g

b. 120 oz = ___?___ lb

There are 16 ounces in a pound.

120 oz ÷ 16 = 7.5 lb

• Examples c and d involve two-step conversions.

c. 5.47 kg = ___?___ mg

There are 1000 milligrams in a gram.
Change *kilograms* to *grams*.

5.47 kg = ___?___ g

5.47 kg × 1000 = 5470 g

Then change *grams* to *milligrams*.

5470 g = ___?___ mg

5470 g × 1000 = 5,470,000 mg

d. 5 T = ___?___ oz

There are 16 ounces in a pound.
Change *tons* to *pounds*.

5 T = ___?___ lb

5 T × 2000 = 10,000 lb

Then change *pounds* to *ounces*.

10,000 lb = ___?___ oz

10,000 lb × 16 = 160,000 oz

Exercises **State which metric unit you would probably use to measure each item.**

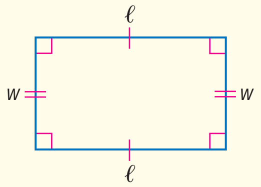
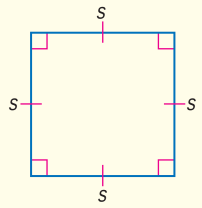
1. radius of a tennis ball
2. length of a notebook
3. mass of a textbook
4. mass of a beach ball
5. width of a football field
6. thickness of a penny
7. amount of liquid in a cup
8. amount of water in a bath tub

Complete each sentence.

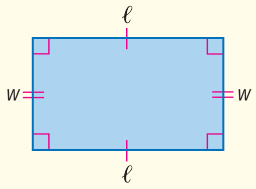
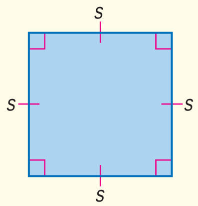
9. 120 in. = ___?___ ft
10. 18 ft = ___?___ yd
11. 10 km = ___?___ m
12. 210 mm = ___?___ cm
13. 180 mm = ___?___ m
14. 3100 m = ___?___ km
15. 90 in. = ___?___ yd
16. 5280 yd = ___?___ mi
17. 8 yd = ___?___ ft
18. 0.62 km = ___?___ m
19. 370 mL = ___?___ L
20. 12 L = ___?___ mL
21. 32 fl oz = ___?___ c
22. 5 qt = ___?___ c
23. 10 pt = ___?___ qt
24. 48 c = ___?___ gal
25. 4 gal = ___?___ qt
26. 36 mg = ___?___ g
27. 13 lb = ___?___ oz
28. 130 g = ___?___ kg
29. 9.05 kg = ___?___ g

Perimeter and Area of Rectangles and Squares

Perimeter is the distance around a figure whose sides are segments. Perimeter is measured in linear units.

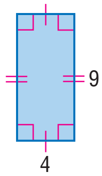
Perimeter of a Rectangle		Perimeter of a Square	
Words	Multiply two times the sum of the length and width.	**Words**	Multiply 4 times the length of a side.
Formula	$P = 2(\ell + w)$	**Formula**	$P = 4s$
			

Area is the number of square units needed to cover a surface. Area is measured in square units.

Area of a Rectangle		Area of a Square	
Words	Multiply the length and width.	**Words**	Square the length of a side.
Formula	$A = \ell w$	**Formula**	$A = s^2$
			

Example 1 **Find the perimeter and area of each rectangle.**

a.



$P = 2(\ell + w)$ Perimeter formula

$= 2(4 + 9)$ Replace ℓ with 4 and w with 9.

$= 26$ Simplify.

$A = \ell w$ Area formula

$= 4 \cdot 9$ Replace ℓ with 4 and w with 9.

$= 36$ Multiply.

The perimeter is 26 units, and the area is 36 square units.

b. a rectangle with length 8 units and width 3 units.

$P = 2(\ell + w)$ Perimeter formula

$= 2(8 + 3)$ Replace ℓ with 8 and w with 3.

$= 22$ Simplify.

$A = \ell \cdot w$ Area formula

$= 8 \cdot 3$ Replace ℓ with 8 and w with 3.

$= 24$ Multiply

The perimeter is 22 units, and the area is 24 square units.

Example 2 **Find the perimeter and area of a square that has a side of length 14 feet.**

$P = 4s$ Perimeter formula

$= 4(14)$ $s = 14$

$= 56$ Multiply.

$A = s^2$ Area formula

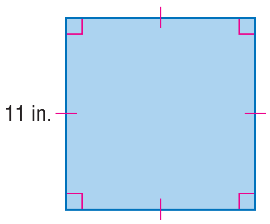
$= 14^2$ $s = 14$

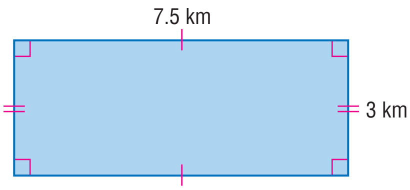
$= 196$ Multiply.

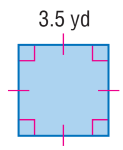
The perimeter is 56 feet, and the area is 196 square feet.

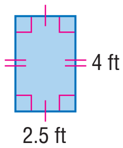
Exercises

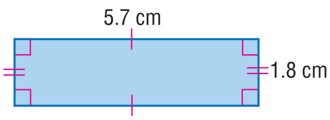
Find the perimeter and area of each figure.

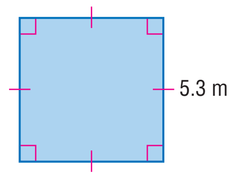
1. 

2. 

3. 

4. 

5. 

6. 

7. a rectangle with length 7 meters and width 11 meters

8. a square with length 4.5 inches

9. a rectangular sandbox with length 2.4 meters and width 1.6 meters

10. a square with length 6.5 yards

11. a square office with length 12 feet

12. a rectangle with length 4.2 inches and width 15.7 inches

13. a square with length 18 centimeters

14. a rectangle with length 5.3 feet and width 7 feet

15. **FENCING** Jansen purchased a lot that was 121 feet in width and 360 feet in length. If he wants to build a fence around the entire lot, how many feet of fence does he need?

16. **CARPETING** Leonardo's bedroom is 10 feet wide and 11 feet long. If the carpet store has a remnant whose area is 105 square feet, could it be used to cover his bedroom floor? Explain.

Operations with Integers

- The absolute value of any number n is its distance from zero on a number line and is written as $|n|$. Since distance cannot be less than zero, the absolute value of a number is always greater than or equal to zero.

Example 1 **Evaluate each expression.**

a. $|3|$

$|3| = 3$ Definition of absolute value

b. $|-7|$

$|-7| = 7$ Definition of absolute value

c. $|-4 + 2|$

$|-4 + 2| = |-2|$ $-4 + 2 = -2$

$= 2$ Simplify.

- To add integers with the same sign, add their absolute values. Give the result the same sign as the integers. To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

Example 2 **Find each sum.**

a. $-3 + (-5)$ Both numbers are negative, so the sum is negative.

$-3 + (-5) = -8$ Add $|-3|$ and $|-5|$.

b. $-4 + 2$ The sum is negative because $|-4| > |2|$.

$-4 + 2 = -2$ Subtract $|2|$ from $|-4|$.

c. $6 + (-3)$ The sum is positive because $|6| > |-3|$.

$6 + (-3) = 3$ Subtract $|-3|$ from $|6|$.

d. $1 + 8$ Both numbers are positive, so the sum is positive

$1 + 8 = 9$ Add $|1|$ and $|8|$.

- To subtract an integer, add its additive inverse.

Example 3 **Find each difference.**

a. $4 - 7$

$4 - 7 = 4 + (-7)$ To subtract 7, add -7.

$= -3$

b. $2 - (-4)$

$2 - (-4) = 2 + 4$ To subtract -4, add 4.

$= 6$

- The product of two integers with different signs is negative. The product of two integers with the same sign is positive. Similarly, the quotient of two integers with different signs is negative, and the quotient of two integers with the same sign is positive.

Prerequisite Skills

Example 4 **Find each product or quotient.**

a. $4(-7)$ — The factors have different signs.

$4(-7) = -28$ — The product is negative.

b. $-64 \div (-8)$ — The dividend and divisor have the same sign.

$-64 \div (-8) = 8$ — The quotient is positive.

c. $-9(-6)$ — The factors have the same sign.

$-9(-6) = 54$ — The product is positive.

d. $-55 \div 5$ — The dividend and divisor have different signs.

$-55 \div 5 = -11$ — The quotient is negative.

e. $\frac{24}{-3}$ — The dividend and divisor have different signs.

$\frac{24}{-3} = -8$ — The quotient is negative.

- To evaluate expressions with absolute value, evaluate the absolute values first and then perform the operation.

Example 5 **Evaluate each expression.**

a. $|-3| - |5|$

$|-3| - |5| = 3 - 5$ — $|-3| = 3$, $|5| = 5$

$= -2$ — Simplify.

b. $|-5| + |-2|$

$|-5| + |-2| = 5 + 2$ — $|-5| = 5$, $|-2| = 2$

$= 7$ — Simplify.

Exercises **Evaluate each absolute value.**

1. $|-3|$ **2.** $|4|$ **3.** $|0|$ **4.** $|-5|$

Find each sum or difference.

5. $-4 - 5$ **6.** $3 + 4$ **7.** $9 - 5$ **8.** $-2 - 5$

9. $3 - 5$ **10.** $-6 + 11$ **11.** $-4 + (-4)$ **12.** $5 - 9$

13. $-3 + 1$ **14.** $-4 + (-2)$ **15.** $2 - (-8)$ **16.** $7 + (-3)$

17. $-4 - (-2)$ **18.** $3 - (-3)$ **19.** $3 + (-4)$ **20.** $-3 - (-9)$

Evaluate each expression.

21. $|-4| - |6|$ **22.** $|-7| + |-1|$ **23.** $|1| + |-2|$ **24.** $|2| - |-5|$

25. $|-5 + 2|$ **26.** $|6 + 4|$ **27.** $|3 - 7|$ **28.** $|-3 - 3|$

Find each product or quotient.

29. $-36 \div 9$ **30.** $-3(-7)$ **31.** $6(-4)$ **32.** $-25 \div 5$

33. $-6(-3)$ **34.** $7(-8)$ **35.** $-40 \div (-5)$ **36.** $11(3)$

37. $44 \div (-4)$ **38.** $-63 \div (-7)$ **39.** $6(5)$ **40.** $-7(12)$

41. $-10(4)$ **42.** $80 \div (-16)$ **43.** $72 \div 9$ **44.** $39 \div 3$

Prerequisite Skills

Evaluating Algebraic Expressions

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

Order of Operations	
Step 1	Evaluate expressions inside grouping symbols.
Step 2	Evaluate all powers.
Step 3	Do all multiplications and/or divisions from left to right.
Step 4	Do all additions and/or subtractions from left to right.

Example 1 **Evaluate each expression.**

a. $x - 5 + y$ **if** $x = 15$ **and** $y = -7$

$x - 5 + y = 15 - 5 + (-7)$ — $x = 15, y = -7$

$= 10 + (-7)$ — Subtract 5 from 15.

$= 3$ — Add.

b. $6ab^2$ **if** $a = -3$ **and** $b = 3$

$6ab^2 = 6(-3)(3)^2$ — $a = -3, b = 3$

$= 6(-3)(9)$ — $3^2 = 9$

$= (-18)(9)$ — Multiply.

$= -162$ — Multiply.

Example 2 **Evaluate each expression if** $m = -2$, $n = -4$, **and** $p = 5$.

a. $\frac{2m + n}{p - 3}$

The division bar is a grouping symbol. Evaluate the numerator and denominator before dividing.

$\frac{2m + n}{p - 3} = \frac{2(-2) + (-4)}{5 - 3}$ — Replace m with -2, n with -4, and p with 5.

$= \frac{-4 - 4}{5 - 3}$ — Multiply.

$= \frac{-8}{2}$ — Subtract.

$= -4$ — Simplify.

b. $-3(m^2 + 2n)$

$-3(m^2 + 2n) = -3[(-2)^2 + 2(-4)]$ — Replace m with -2 and n with -4.

$= -3[4 + (-8)]$ — Multiply.

$= -3(-4)$ — Add.

$= 12$ — Multiply.

Example 3 **Evaluate** $3|a - b| + 2|c - 5|$ **if** $a = -2$, $b = -4$, **and** $c = 3$.

$3|a - b| + 2|c - 5| = 3|-2 - (-4)| + 2|3 - 5|$ — Substitute for a, b, and c.

$= 3|2| + 2|-2|$ — Simplify.

$= 3(2) + 2(2)$ — Find absolute values.

$= 10$ — Simplify.

Exercises

Evaluate each expression if $a = 2$, $b = -3$, $c = -1$, **and** $d = 4$.

1. $2a + c$
2. $\frac{bd}{2c}$
3. $\frac{2d - a}{b}$
4. $3d - c$
5. $\frac{3b}{5a + c}$
6. $5bc$
7. $2cd + 3ab$
8. $\frac{c - 2d}{a}$

Evaluate each expression if $x = 2$, $y = -3$, **and** $z = 1$.

9. $24 + |x - 4|$
10. $13 + |8 + y|$
11. $|5 - z| + 11$
12. $|2y - 15| + 7$
13. $|y| - 7$
14. $11 - 7 + |-x|$
15. $|x| - |2z|$
16. $|z - y| + 6$

Solving Linear Equations

- If the same number is added to or subtracted from each side of an equation, the resulting equation is true.

Example 1 **Solve each equation.**

a. $x - 7 = 16$

$x - 7 = 16$ Original equation

$x - 7 + 7 = 16 + 7$ Add 7 to each side.

$x = 23$ Simplify.

b. $m + 12 = -5$

$m + 12 = -5$ Original equation

$m + 12 + (-12) = -5 + (-12)$ Add -12 to each side.

$m = -17$ Simplify.

c. $k + 31 = 10$

$k + 31 = 10$ Original equation

$k + 31 - 31 = 10 - 31$ Subtract 31 from each side.

$k = -21$ Simplify.

- If each side of an equation is multiplied or divided by the same number, the resulting equation is true.

Example 2 **Solve each equation.**

a. $4d = 36$

$4d = 36$ Original equation

$\frac{4d}{4} = \frac{36}{4}$ Divide each side by 4.

$x = 9$ Simplify.

b. $-\frac{t}{8} = -7$

$-\frac{t}{8} = -7$ Original equation.

$-8\left(-\frac{t}{8}\right) = -8(-7)$ Multiply each side by -8.

$t = 56$ Simplify.

c. $\frac{3}{5}x = -8$

$\frac{3}{5}x = -8$ Original equation.

$\frac{5}{3}\left(\frac{3}{5}x\right) = \frac{5}{3}(-8)$ Multiply each side by $\frac{5}{3}$.

$x = -\frac{40}{3}$ Simplify.

- To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.

Example 3 **Solve each equation.**

a. $12 - m = 20$

$12 - m = 20$ Original equation

$12 - m - 12 = 20 - 12$ Subtract 12 from each side.

$-m = 8$ Simplify.

$m = -8$ Divide each side by -1.

b. $8q - 15 = 49$

$8q - 15 = 49$	Original equation
$8q - 15 + 15 = 49 + 15$	Add 15 to each side.
$8q = 64$	Simplify.
$\frac{8q}{8} = \frac{64}{8}$	Divide each side by 8.
$q = 8$	Simplify.

c. $12y + 8 = 6y - 5$

$12y + 8 = 6y - 5$	Original equation
$12y + 8 - 8 = 6y - 5 - 8$	Subtract 8 from each side.
$12y = 6y - 13$	Simplify.
$12y - 6y = 6y - 13 - 6y$	Subtract 6y from each side.
$6y = -13$	Simplify.
$\frac{6y}{6} = \frac{-13}{6}$	Divide each side by 6.
$y = -\frac{13}{6}$	Simplify.

- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 4 **Solve $3(x - 5) = 13$.**

$3(x - 5) = 13$	Original equation
$3x - 15 = 13$	Distributive Property
$3x - 15 + 15 = 13 + 15$	Add 15 to each side.
$3x = 28$	Simplify.
$x = \frac{28}{3}$	Divide each side by 3.

Exercises Solve each equation.

1. $r + 11 = 3$
2. $n + 7 = 13$
3. $d - 7 = 8$
4. $\frac{8}{5}a = -6$
5. $-\frac{p}{12} = 6$
6. $\frac{x}{4} = 8$
7. $\frac{12}{5}f = -18$
8. $\frac{y}{7} = -11$
9. $\frac{6}{7}y = 3$
10. $c - 14 = -11$
11. $t - 14 = -29$
12. $p - 21 = 52$
13. $b + 2 = -5$
14. $q + 10 = 22$
15. $-12q = 84$
16. $5s = 30$
17. $5c - 7 = 8c - 4$
18. $2\ell + 6 = 6\ell - 10$
19. $\frac{m}{10} + 15 = 21$
20. $-\frac{m}{8} + 7 = 5$
21. $8t + 1 = 3t - 19$
22. $9n + 4 = 5n + 18$
23. $5c - 24 = -4$
24. $3n + 7 = 28$
25. $-2y + 17 = -13$
26. $-\frac{t}{13} - 2 = 3$
27. $\frac{2}{9}x - 4 = \frac{2}{3}$
28. $9 - 4g = -15$
29. $-4 - p = -2$
30. $21 - b = 11$
31. $-2(n + 7) = 15$
32. $5(m - 1) = -25$
33. $-8a - 11 = 37$
34. $\frac{7}{4}q - 2 = -5$
35. $2(5 - n) = 8$
36. $-3(d - 7) = 6$

Solving Inequalities in One Variable

Statements with **greater than** (>), **less than** (<), **greater than or equal to** (≥), or **less than or equal to** (≤) are **inequalities**.

- If any number is added or subtracted to each side of an inequality, the resulting inequality is true.

Example 1 **Solve each inequality.**

a. $x - 17 > 12$

$x - 17 > 12$	Original inequality
$x - 17 + 17 > 12 + 17$	Add 17 to each side.
$x > 29$	Simplify.

The solution set is $\{x \mid x > 29\}$.

b. $y + 11 \leq 5$

$y + 11 \leq 5$	Original inequality
$y + 11 - 11 \leq 5 - 11$	Subtract 11 from each side.
$y \leq -6$	Simplify.

The solution set is $\{y \mid y \leq -6\}$.

- If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.

Example 2 **Solve each inequality.**

a. $\frac{t}{6} \geq 11$

$\frac{t}{6} \geq 11$	Original inequality
$(6)\frac{t}{6} \geq (6)11$	Multiply each side by 6.
$t \geq 66$	Simplify.

The solution set is $\{t \mid t > 66\}$.

b. $8p < 72$

$8p < 72$	Original inequality
$\frac{8p}{8} < \frac{72}{8}$	Divide each side by 8.
$p < 9$	Simplify.

The solution set is $\{p \mid p < 9\}$.

- If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is true.

Example 3 **Solve each inequality.**

a. $-5c > 30$

$-5c > 30$	Original inequality
$\frac{-5c}{-5} < \frac{30}{-5}$	Divide each side by -5. Change > to <.
$c < -6$	Simplify.

The solution set is $\{c \mid c < -6\}$.

b. $-\frac{d}{13} \leq -4$

$-\frac{d}{13} \leq -4$	Original inequality
$(-13)(-\frac{d}{13}) \geq (-13)(-4)$	Multiply each side by -13. Change $\leq$ to $\geq$.
$d \geq 52$	Simplify.

The solution set is $\{d \mid d \geq 52\}$.

- Inequalities involving more than one operation can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 4 **Solve each inequality.**

a. $-6a + 13 < -7$

$-6a + 13 < -7$	Original inequality
$-6a + 13 - 13 < -7 - 13$	Subtract 13 from each side.
$-6a < -20$	Simplify.
$\frac{-6a}{-6} > \frac{-20}{-6}$	Divide each side by -6. Change $<$ to $>$.
$a > \frac{10}{3}$	Simplify.

The solution set is $\left\{a \mid a > \frac{10}{3}\right\}$.

b. $4z + 7 \geq 8z - 1$

$4z + 7 \geq 8z - 1$	Original inequality.
$4z + 7 - 7 \geq 8z - 1 - 7$	Subtract 7 from each side.
$4z \geq 8z - 8$	Simplify.
$4z - 8z \geq 8z - 8 - 8z$	Subtract $8z$ from each side.
$-4z \geq -8$	Simplify.
$\frac{-4z}{-4} \leq \frac{-8}{-4}$	Divide each side by -4. Change $\geq$ to $\leq$.
$z \leq 2$	Simplify.

The solution set is $\{z \mid z \leq 2\}$.

Exercises **Solve each inequality.**

1. $x - 7 < 6$
2. $4c + 23 \leq -13$
3. $-\frac{p}{5} \geq 14$
4. $-\frac{a}{8} < 5$
5. $\frac{t}{6} > -7$
6. $\frac{a}{11} \leq 8$
7. $d + 8 \leq 12$
8. $m + 14 > 10$
9. $2z - 9 < 7z + 1$
10. $6t - 10 \geq 4t$
11. $3z + 8 < 2$
12. $a + 7 \geq -5$
13. $m - 21 < 8$
14. $x - 6 \geq 3$
15. $-3b \leq 48$
16. $4y < 20$
17. $12k \geq -36$
18. $-4h > 36$
19. $\frac{2}{5}b - 6 \leq -2$
20. $\frac{8}{3}t + 1 > -5$
21. $7q + 3 \geq -4q + 25$
22. $-3n - 8 > 2n + 7$
23. $-3w + 1 \leq 8$
24. $-\frac{4}{5}k - 17 > 11$

Graphing Using Intercepts and Slope

- The x-coordinate of the point at which a line crosses the x-axis is called the **x-intercept**. The y-coordinate of the point at which a line crosses the y-axis is called the **y-intercept**. Since two points determine a line, one method of graphing a linear equation is to find these intercepts.

Example 1 **Determine the x-intercept and y-intercept of $4x - 3y = 12$. Then graph the equation.**

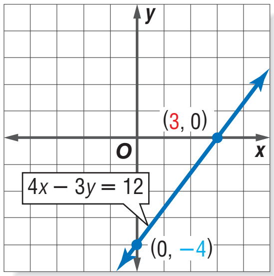
To find the x-intercept, let $y = 0$.

$4x - 3y = 12$	Original equation
$4x - 3(0) = 12$	Replace y with 0.
$4x = 12$	Simplify.
$x = 3$	Divide each side by 4.

To find the y-intercept, let $x = 0$.

$4x - 3y = 12$	Original equation
$4(0) - 3y = 12$	Replace x with 0.
$-3y = 12$	Divide each side by -3.
$y = -4$	Simplify.

Put a point on the x-axis at 3 and a point on the y-axis at -4. Draw the line through the two points.



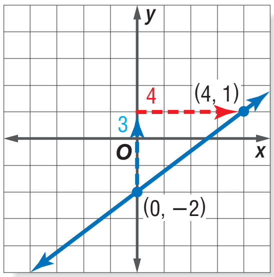
- A linear equation of the form $y = mx + b$ is in *slope-intercept* form, where m is the slope and b is the y-intercept. When an equation is written in this form, you can graph the equation quickly.

Example 2 **Graph $y = \frac{3}{4}x - 2$.**

Step 1 The y-intercept is -2. So, plot a point at $(0, -2)$.

Step 2 The slope is $\frac{3}{4}$. $\frac{\text{rise}}{\text{run}}$
From $(0, -2)$, move up 3 units and right 4 units. Plot a point.

Step 3 Draw a line connecting the points.



Exercises **Graph each equation using both intercepts.**

1. $-2x + 3y = 6$	**2.** $2x + 5y = 10$	**3.** $3x - y = 3$
4. $-x + 2y = 2$	**5.** $3x + 4y = 12$	**6.** $4y + x = 4$

Graph each equation using the slope and y-intercept.

7. $y = -x + 2$	**8.** $y = x - 2$	**9.** $y = x + 1$
10. $y = 3x - 1$	**11.** $y = -2x + 3$	**12.** $y = -3x - 1$

Graph each equation using either method.

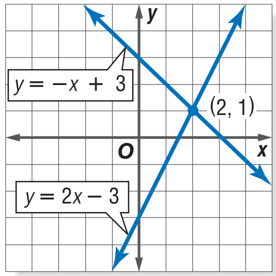
13. $y = \frac{2}{3}x - 3$	**14.** $y = \frac{1}{2}x - 1$	**15.** $y = 2x - 2$
16. $-6x + y = 2$	**17.** $2y - x = -2$	**18.** $3x + 4y = -12$
19. $4x - 3y = 6$	**20.** $4x + y = 4$	**21.** $y = 2x - \frac{3}{2}$

Solving Systems of Linear Equations

- Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

Example 1 **Solve each system of equations by graphing. Then determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.**

a. $y = -x + 3$
$y = 2x - 3$



The graphs appear to intersect at (2, 1). Check this estimate by replacing x with 2 and y with 1 in each equation.

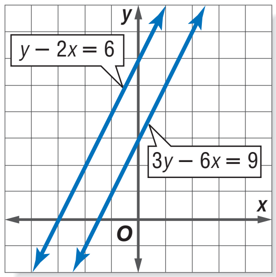
Check: $y = -x + 3$ $y = 2x - 3$

$1 \stackrel{?}{=} -2 + 3$ $1 \stackrel{?}{=} 2(2) - 3$

$1 = 1$ ✓ $1 = 1$ ✓

The system has one solution at (2, 1).

b. $y - 2x = 6$
$3y - 6x = 9$



The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different y-intercepts. Equations with the same slope *and* the same y-intercepts have an infinite number of solutions.

- It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

Example 2 **Use substitution to solve the system of equations.**

$y = -4x$

$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for y in the second equation.

$2y + 3x = 8$	Second equation
$2(-4x) + 3x = 3$	$y = -4x$
$-8x + 3x = 8$	Simplify.
$-5x = 8$	Combine like terms.
$\frac{-5x}{-5} = \frac{8}{-5}$	Divide each side by -5.
$x = -\frac{8}{5}$	Simplify.

Use $y = -4x$ to find the value of y.

$y = -4x$	First equation
$y = -4\left(-\frac{8}{5}\right)$	$x = -\frac{8}{5}$
$y = \frac{32}{5}$	Simplify.

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$.

- Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

Example 3 **Use elimination to solve the system of equations.**

$\mathbf{3x + 5y = 7}$
$\mathbf{4x + 2y = 0}$

Either x or y can be eliminated. In this example, we will eliminate x.

$3x + 5y = 7$	Multiply by 4.	$12x + 20y = 28$
$4x + 2y = 0$	Multiply by -3.	$+\ -12x - 6y = 0$

$$14y = 28 \quad \text{Add the equations.}$$
$$\frac{14y}{14} = \frac{28}{14} \quad \text{Divide each side by 14.}$$
$$y = 2 \quad \text{Simplify.}$$

Now substitute 2 for y in either equation to find the value of x.

$$4x + 2y = 0 \quad \text{Second equation}$$
$$4x + 2(2) = 0 \quad y = 2$$
$$4x + 4 = 0 \quad \text{Simplify.}$$
$$4x + 4 - 4 = 0 - 4 \quad \text{Subtract 4 from each side.}$$
$$4x = -4 \quad \text{Simplify.}$$
$$\frac{4x}{4} = \frac{-4}{4} \quad \text{Divide each side by 4.}$$
$$x = -1 \quad \text{Simplify.}$$

The solution is $(-1, 2)$.

Exercises **Solve by graphing.**

1. $y = -x + 2$; $y = -\frac{1}{2}x + 1$
2. $y = 3x - 3$; $y = x + 1$
3. $y - 2x = 1$; $2y - 4x = 1$
4. $2x - 4y = -2$; $-6x + 12y = 6$
5. $4x + 3y = 12$; $3x - y = 9$
6. $3y + x = -3$; $y - 3x = -1$

Solve by substitution.

7. $-5x + 3y = 12$; $x + 2y = 8$
8. $x - 4y = 22$; $2x + 5y = -21$
9. $y + 5x = -3$; $3y - 2x = 8$
10. $y - 2x = 2$; $7y + 4x = 23$
11. $2x - 3y = -8$; $-x + 2y = 5$
12. $4x + 2y = 5$; $3x - y = 10$

Solve by elimination.

13. $-3x + y = 7$; $3x + 2y = 2$
14. $3x + 4y = -1$; $-9x - 4y = 13$
15. $-4x + 5y = -11$; $2x + 3y = 11$
16. $6x - 5y = 1$; $-2x + 9y = 7$
17. $3x - 2y = 8$; $5x - 3y = 16$
18. $4x + 7y = -17$; $3x + 2y = -3$

Name an appropriate method to solve each system of equations. Then solve the system.

19. $4x - y = 11$; $2x - 3y = 3$
20. $4x + 6y = 3$; $-10x - 15y = -4$
21. $3x - 2y = 6$; $5x - 5y = 5$
22. $3y + x = 3$; $-2y + 5x = 15$
23. $4x - 7y = 8$; $-2x + 5y = -1$
24. $x + 3y = 6$; $4x - 2y = -32$

Square Roots and Simplifying Radicals

- A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.
- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.
- The **Product Property** states that for two numbers a and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1 **Simplify.**

a. $\sqrt{45}$

$$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5} \quad \text{Prime factorization of 45}$$
$$= \sqrt{3^2} \cdot \sqrt{5} \quad \text{Product Property of Square Roots}$$
$$= 3\sqrt{5} \quad \text{Simplify.}$$

b. $\sqrt{3} \cdot \sqrt{3}$

$$\sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3} \quad \text{Product Property}$$
$$= \sqrt{9} \text{ or } 3 \quad \text{Simplify.}$$

c. $\sqrt{6} \cdot \sqrt{15}$

$$\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15} \quad \text{Product Property}$$
$$= \sqrt{3 \cdot 2 \cdot 3 \cdot 5} \quad \text{Prime factorization}$$
$$= \sqrt{3^2} \cdot \sqrt{10} \quad \text{Product Property}$$
$$= 3\sqrt{10} \quad \text{Simplify.}$$

- For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

Example 2 $\sqrt{20x^3y^5z^6}$

$$\sqrt{20x^3y^5z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6} \quad \text{Prime factorization}$$
$$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6} \quad \text{Product Property}$$
$$= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3| \quad \text{Simplify.}$$
$$= 2xy^2|z^3|\sqrt{5xy} \quad \text{Simplify.}$$

- The **Quotient Property** states that for any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example 3 **Simplify** $\sqrt{\frac{25}{16}}$.

$$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} \quad \text{Quotient Property}$$
$$= \frac{5}{4} \quad \text{Simplify.}$$

- Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

Example 4 **Simplify.**

a. $\frac{2}{\sqrt{3}}$

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}}.$$

$$= \frac{2\sqrt{3}}{3} \quad \text{Simplify.}$$

b. $\frac{\sqrt{13y}}{\sqrt{18}}$

$$\frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}} \quad \text{Prime factorization}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \quad \text{Product Property}$$

$$= \frac{\sqrt{13y}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \quad \text{Multiply by } \frac{\sqrt{2}}{\sqrt{2}}.$$

$$= \frac{\sqrt{26y}}{6} \quad \text{Product Property}$$

- Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$.

Example 5 **Simplify** $\frac{3}{5 - \sqrt{2}}$.

$$\frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}} \quad \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$$

$$= \frac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{15 + 3\sqrt{2}}{25 - 2} \quad \text{Multiply. } (\sqrt{2})^2 = 2$$

$$= \frac{15 + 3\sqrt{2}}{23} \quad \text{Simplify.}$$

Exercises **Simplify.**

1. $\sqrt{32}$
2. $\sqrt{75}$
3. $\sqrt{50} \cdot \sqrt{10}$
4. $\sqrt{12} \cdot \sqrt{20}$
5. $\sqrt{6} \cdot \sqrt{6}$
6. $\sqrt{16} \cdot \sqrt{25}$
7. $\sqrt{98x^3y^6}$
8. $\sqrt{56a^2b^4c^5}$
9. $\sqrt{\frac{81}{49}}$
10. $\sqrt{\frac{121}{16}}$
11. $\sqrt{\frac{63}{8}}$
12. $\sqrt{\frac{288}{147}}$
13. $\frac{\sqrt{10p^3}}{\sqrt{27}}$
14. $\frac{\sqrt{108}}{\sqrt{2q^6}}$
15. $\frac{4}{5 - 2\sqrt{3}}$
16. $\frac{7\sqrt{3}}{5 - 2\sqrt{6}}$
17. $\frac{3}{\sqrt{48}}$
18. $\frac{\sqrt{24}}{\sqrt{125}}$
19. $\frac{3\sqrt{5}}{2 - \sqrt{2}}$
20. $\frac{3}{-2 + \sqrt{13}}$

Multiplying Polynomials

- The **Product of Powers** rule states that for any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify each expression.**

a. $(4p^5)(p^4)$

$$\begin{aligned}(4p^5)(p^4) &= (4)(1)(p^5 \cdot p^4) && \text{Commutative and Associative Properties}\\ &= (4)(1)(p^{5+4}) && \text{Product of powers}\\ &= 4p^9 && \text{Simplify.}\end{aligned}$$

b. $(3yz^5)(-9y^2z^2)$

$$\begin{aligned}(3yz^5)(-9y^2z^2) &= (3)(-9)(y \cdot y^2)(z^5 \cdot z^2) && \text{Commutative and Associative Properties}\\ &= -27(y^{1+2})(z^{5+2}) && \text{Product of powers}\\ &= -27y^3z^7 && \text{Simplify.}\end{aligned}$$

- The Distributive Property can be used to multiply a monomial by a polynomial.

Example 2 **Simplify $3x^3(-4x^2 + x - 5)$.**

$$\begin{aligned}3x^3(-4x^2 + x - 5) &= 3x^3(-4x^2) + 3x^3(x) - 3x^3(5) && \text{Distributive Property}\\ &= -12x^5 + 3x^4 - 15x^3 && \text{Multiply.}\end{aligned}$$

- To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

Example 3 **Simplify each expression.**

a. $(-3x^2y^4)^3$

$$\begin{aligned}(-3x^2y^4)^3 &= (-3)^3(x^2)^3(y^4)^3 && \text{Power of a product}\\ &= -27x^6y^{12} && \text{Power of a power}\end{aligned}$$

b. $(xy)^3(-2x^4)^2$

$$\begin{aligned}(xy)^3(-2x^4)^2 &= x^3y^3(-2)^2(x^4)^2 && \text{Power of a product}\\ &= x^3y^3(4)x^8 && \text{Power of a power}\\ &= 4x^3 \cdot x^8 \cdot y^3 && \text{Commutative Property}\\ &= 4x^{11}y^3 && \text{Product of powers}\end{aligned}$$

- To multiply two binomials, find the sum of the products of

F the *First* terms,
O the *Outer* terms,
I the *Inner* terms, and
L the *Last* terms.

Example 4 **Find each product.**

a. $(2x - 3)(x + 1)$

$$\begin{aligned}(2x - 3)(x + 1) &= \overset{F}{(2x)(x)} + \overset{O}{(2x)(1)} + \overset{I}{(-3)(x)} + \overset{L}{(-3)(1)} && \text{FOIL method}\\ &= 2x^2 + 2x - 3x - 3 && \text{Multiply.}\\ &= 2x^2 - x - 3 && \text{Combine like terms.}\end{aligned}$$

b. $(x + 6)(x + 5)$

$$\begin{aligned}(x + 6)(x + 5) &= \overset{F}{(x)(x)} + \overset{O}{(x)(5)} + \overset{I}{(6)(x)} + \overset{L}{(6)(5)} && \text{FOIL method}\\ &= x^2 + 5x + 6x + 30 && \text{Multiply.}\\ &= x^2 + 11x + 30 && \text{Combine like terms.}\end{aligned}$$

- The Distributive Property can be used to multiply any two polynomials.

Example 5 **Find $(3x - 2)(2x^2 + 7x - 4)$.**

$$(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4) \quad \text{Distributive Property}$$
$$= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8 \quad \text{Distributive Property}$$
$$= 6x^3 + 17x^2 - 26x + 8 \quad \text{Combine like terms.}$$

- Three special products are: $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$, and $(a + b)(a - b) = a^2 - b^2$.

Example 6 **Find each product.**

a. $(2x - z)^2$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a difference}$$
$$(2x - z)^2 = (2x)^2 - 2(2x)(z) + (z)^2 \quad a = 2x \text{ and } b = z$$
$$= 4x^2 - 4xz + z^2 \quad \text{Simplify.}$$

b. $(3x + 7)(3x - 7)$

$$(a + b)(a - b) = a^2 - b^2 \quad \text{Product of sum and difference}$$
$$(3x + 7)(3x - 7) = (3x)^2 - (7)^2 \quad a = 3x \text{ and } b = 7$$
$$= 9x^2 - 49 \quad \text{Simplify.}$$

Exercises **Find each product.**

1. $(3q^2)(q^5)$
2. $(5m)(4m^3)$
3. $\left(\frac{9}{2}c\right)\left(8c^5\right)$
4. $(n^6)(10n^2)$
5. $(fg^8)(15f^2g)$
6. $(6j^4k^4)(j^2k)$
7. $(2ab^3)(4a^2b^2)$
8. $\left(\frac{8}{5}x^3y\right)\left(4x^3y^2\right)$
9. $-2q^2(q^2 + 3)$
10. $5p(p - 18)$
11. $15c(-3c^2 + 2c + 5)$
12. $8x(-4x^2 - x + 11)$
13. $4m^2(-2m^2 + 7m - 5)$
14. $8y^2(5y^3 - 2y + 1)$
15. $\left(\frac{3}{2}m^3n^2\right)^2$
16. $(-2c^3d^2)^2$
17. $(-5wx^5)^3$
18. $(6a^5b)^3$
19. $(k^2\ell)^3(13k^2)^2$
20. $(-5w^3x^2)^2(2w^5)^2$
21. $(-7y^3z^2)(4y^2)^4$
22. $\left(\frac{1}{2}p^2q^2\right)^2\left(4pq^3\right)^3$
23. $(m - 1)(m - 4)$
24. $(s - 7)(s - 2)$
25. $(x - 3)(x + 4)$
26. $(a + 3)(a - 6)$
27. $(5d + 3)(d - 4)$
28. $(q + 2)(3q + 5)$
29. $(2q + 3)(5q + 2)$
30. $(2a - 3)(2a - 5)$
31. $(d + 1)(d - 1)$
32. $(4a - 3)(4a + 3)$
33. $(s - 5)^2$
34. $(3f - g)^2$
35. $(2r - 5)^2$
36. $\left(t + \frac{8}{3}\right)^2$
37. $(x + 4)(x^2 - 5x - 2)$
38. $(x - 2)(x^2 + 3x - 7)$
39. $(3b - 2)(3b^2 + b + 1)$
40. $(2j + 7)(j^2 - 2j + 4)$

Dividing Polynomials

- The **Quotient of Powers** rule states that for any nonzero number a and all integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.

Example 1 **Simplify.**

a. $\frac{x^5y^8}{-xy^3}$

$$\frac{x^5y^8}{-xy^3} = \left(\frac{x^5}{-x}\right)\left(\frac{y^8}{y^3}\right)$$ Group powers that have the same base.

$$= -(x^{5-1})(y^{8-3})$$ Quotient of powers

$$= -x^4y^5$$ Simplify.

b. $\left(\frac{4z^3}{3}\right)^3$

$$\left(\frac{4z^3}{3}\right)^3 = \frac{(4z^3)^3}{3^3}$$ Power of a quotient

$$= \frac{4^3(z^3)^3}{3^3}$$ Power of a product

$$= \frac{64z^9}{27}$$ Power of a product

c. $\frac{w^{-2}x^4}{2w^{-5}}$

$$\frac{w^{-2}x^4}{2w^{-5}} = \frac{1}{2}\left(\frac{w^{-2}}{w^{-5}}\right)x^4$$ Group powers that have the same base.

$$= \frac{1}{2}(w^{-2-(-5)})x^4$$ Quotient of powers

$$= \frac{1}{2}w^3x^4$$ Simplify.

- You can divide a polynomial by a monomial by separating the terms of the numerator.

Example 2 **Simplify $\frac{15x^3 - 3x^2 + 12x}{3x}$.**

$$\frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x}$$ Divide each term by $3x$.

$$= 5x^2 - x + 4$$ Simplify.

- Division can sometimes be performed using factoring.

Example 3 **Find $(n^2 - 8n - 9) \div (n - 9)$.**

$$(n^2 - 8n - 9) \div (n - 9) = \frac{n^2 - 8n - 9}{(n - 9)}$$ Write as a rational expression.

$$= \frac{(n - 9)(n + 1)}{(n - 9)}$$ Factor the numerator.

$$= \frac{\cancel{(n - 9)}(n + 1)}{\cancel{(n - 9)}}$$ Divide by the GCF.

$$= n + 1$$ Simplify.

- When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

Example 4 **Find $(n^3 - 4n^2 - 9) \div (n - 3)$.**

In this case, there is no n term, so you must rename the dividend using 0 as the coefficient of the missing term.

$(n^3 - 4n^2 + 9) \div (n - 3) = (n^3 - 4n^2 + 0n + 9) \div (n - 3)$

Divide the first term of the dividend, n^3, by the first term of the divisor, n.

$$\begin{array}{rll} & n^2 - n - 3 & \\ n - 3 \enclose{longdiv}{} & n^3 - 4n^2 + 0n + 12 & \\ & \underline{(-)\, n^3 - 3n^2} & \text{Multiply } n^2 \text{ and } n - 3. \\ & -n^2 + 0n & \text{Subtract and bring down } 0n. \\ & \underline{(-) -n^2 + 3n} & \text{Multiply } -n \text{ and } n - 3. \\ & -3n + 12 & \text{Subtract and bring down 12.} \\ & \underline{(-) -3n + 9} & \text{Multiply } -3 \text{ and } n - 3. \\ & 3 & \text{Subtract.} \end{array}$$

Therefore, $(n^3 - 4n^2 + 9) \div (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}$. Since the quotient has a nonzero remainder, $n - 3$ is not a factor of $n^3 - 4n^2 + 9$.

Exercises **Find each quotient.**

1. $\frac{a^2c^2}{2a}$
2. $\frac{5q^5r^3}{q^2r^2}$
3. $\frac{b^2d^5}{8b^{-2}d^3}$
4. $\frac{5p^{-3}x}{2p^{-7}}$
5. $\frac{3r^{-3}s^2t^4}{2r^2st^{-3}}$
6. $\frac{3x^3y^{-1}z^5}{xyz^2}$
7. $\left(\frac{w^4}{6}\right)^3$
8. $\left(\frac{-3q^2}{5}\right)^3$
9. $\left(\frac{-2y^2}{7}\right)^2$
10. $\left(\frac{5m^2}{3}\right)^4$
11. $\frac{4z^2 - 16z - 36}{4z}$
12. $(5d^2 + 8d - 20) \div 10d$
13. $(p^3 - 12p^2 + 3p + 8) \div 4p$
14. $(b^3 + 4b^2 + 10) \div 2b$
15. $\frac{a^3 - 6a^2 + 4a - 3}{a^2}$
16. $\frac{8x^2y - 10xy^2 + 6x^3}{2x^2}$
17. $\frac{s^2 - 2s - 8}{s - 4}$
18. $(r^2 + 9r + 20) \div (r + 5)$
19. $(t^2 - 7t + 12) \div (t - 3)$
20. $(c^2 + 3c - 54) \div (c + 9)$
21. $(2p^2 - 9p - 5) \div (q - 5)$
22. $\frac{3z^2 - 2z - 5}{z + 1}$
23. $\frac{(m^3 + 3m^2 - 5m + 1)}{m - 1}$
24. $(d^3 - 2d^2 + 4d + 24) \div (d + 2)$
25. $(2j^3 + 5j + 26) \div (j + 2)$
26. $\frac{2x^3 + 3x^2 - 176}{x - 4}$
27. $(x^2 + 6x - 3) \div (x + 4)$
28. $\frac{h^3 + 2h^2 - 6h + 1}{h - 2}$

Prerequisite Skills

Factoring to Solve Equations

- Some polynomials can be factored using the Distributive Property.

Example 1 **Factor $5t^2 + 15t$.**

Find the greatest common factor (GCF) of $5t^2$ and $15t$.

$5t^2 = 5 \cdot t \cdot t$, $15t = 3 \cdot 5 \cdot t$ — GCF: $5 \cdot t$ or $5t$

$5t^2 + 15t = 5t(t) + 5t(3)$ — Rewrite each term using the GCF.

$= 5t(t + 3)$ — Distributive Property

- To factor polynomials of the form $x^2 + bx + c$, find two integers m and n so that $mn = c$ and $m + n = b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

Example 2 **Factor each polynomial.**

a. $x^2 + 7x + 10$ — Both b and c are positive.

In this equation, b is 7 and c is 10. Find two numbers with a product of 10 and with a sum of 7.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

$x^2 + 7x + 10 = (x + m)(x + n)$ — The correct factors are 2 and 5.

$= (x + 2)(x + 5)$ — Write the pattern; $m = 2$ and $n = 5$.

b. $x^2 - 8x + 15$ — b is negative and c is positive.

In this equation, b is -8 and c is 15. This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

Factors of 15	Sum of Factors
−1, −15	−16
−3, −5	−8

$x^2 - 8x + 15 = (x + m)(x + n)$ — The correct factors are -3 and -5.

$= (x - 3)(x - 5)$ — Write the pattern; $m = -3$ and $n = -5$.

- To factor polynomials of the form $ax^2 + bx + c$, find two integers m and n with a product equal to ac and with a sum equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

c. $5x^2 - 19x - 4$ — b is negative and c is negative.

In this equation, a is 5, b is -19, and c is -4.
Find two numbers with a product of -20 and with a sum of -19.

Factors of −20	Sum of Factors
−2, 10	8
2, −10	−8
−1, 20	19
1, −20	−19

The correct factors are 1 and -20.

$5x^2 - 19x - 4 = 5x^2 + mx + nx - 4$ — Write the pattern.

$= 5x^2 + x + (-20)x - 4$ — $m = 1$ and $n = -20$

$= (5x^2 + x) - (20x + 4)$ — Group terms with common factors.

$= x(5x + 1) - 4(5x + 1)$ — Factor the GCF from each group.

$= (x - 4)(5x + 1)$ — Distributive Property

- Here are some special products.

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 3 **Factor each polynomial.**

a. $9x^2 + 6x + 1$ ← The first and last terms are perfect squares, and the middle term is equal to $2(3x)(1)$.

$9x^2 + 6x + 1 = (3x)^2 + 2(3x)(1) + 1^2$ Write as $a^2 + 2ab + b^2$.

$= (3x + 1)^2$ Factor using the pattern.

b. $x^2 - 9 = 0$ ← This is a difference of squares.

$x^2 - 9 = x^2 - (3)^2$ Write in the form $a^2 - b^2$.

$= (x - 3)(x + 3)$ Factor the difference of squares.

- The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$. Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

Example 4 **Solve $x^2 - 5x + 4 = 0$ by factoring.**

Factor the polynomial. This expression is of the form $x^2 + bx + c$.

$x^2 - 5x + 4 = 0$ Original equation

$(x - 1)(x - 4) = 0$ Factor the polynomial.

If $ab = 0$, then $a = 0$, $b = 0$, or both equal 0. Let each factor equal 0.

$x - 1 = 0$ or $x - 4 = 0$

$x = 1$ $x = 4$

Exercises Factor each polynomial.

1. $u^2 - 12u$
2. $w^2 + 4w$
3. $7j^2 - 28j$
4. $2g^2 + 24g$
5. $6x^2 + 2x$
6. $5t^2 - 30t$
7. $z^2 + 10z + 21$
8. $n^2 + 8n + 15$
9. $h^2 + 8h + 12$
10. $x^2 + 14x + 48$
11. $m^2 + 6m - 7$
12. $b^2 + 2b - 24$
13. $q^2 - 9q + 18$
14. $p^2 - 5p + 6$
15. $a^2 - 3a - 4$
16. $k^2 - 4k - 32$
17. $n^2 - 7n - 44$
18. $y^2 - 3y - 88$
19. $3z^2 + 4z - 4$
20. $2y^2 + 9y - 5$
21. $5x^2 + 7x + 2$
22. $3s^2 + 11s - 4$
23. $6r^2 - 5r + 1$
24. $8a^2 + 15a - 2$
25. $w^2 - \frac{9}{4}$
26. $c^2 - 64$
27. $r^2 + 14r + 49$
28. $b^2 + 18b + 81$
29. $j^2 - 12j + 36$
30. $4t^2 - 25$

Solve each equation by factoring.

31. $10r^2 - 35r = 0$
32. $3x^2 + 15x = 0$
33. $k^2 + 13k + 36 = 0$
34. $w^2 - 8w + 12 = 0$
35. $c^2 - 5c - 14 = 0$
36. $z^2 - z - 42 = 0$
37. $2y^2 - 5y - 12 = 0$
38. $3b^2 - 4b - 15 = 0$
39. $t^2 + 12t + 36 = 0$
40. $u^2 + 5u + \frac{25}{4} = 0$
41. $q^2 - 8q + 16 = 0$
42. $a^2 - 6a + 9 = 0$

Operations with Matrices

- A **matrix** is a rectangular arrangement of numbers in rows and columns. Each entry in a matrix is called an **element**. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first.

- For example, matrix A has dimensions 3×2 and matrix B has dimensions 2×4.

$$\text{matrix } A = \begin{bmatrix} 6 & -2 \\ 0 & 5 \\ -4 & 10 \end{bmatrix} \qquad \text{matrix } B = \begin{bmatrix} 7 & -1 & -2 & 0 \\ 3 & 6 & -5 & 2 \end{bmatrix}$$

- If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

Example 1 **If $A = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix}$, and $C = \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix}$, find the sum and difference.**

a. $A + B$

$$A + B = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 12 + (-3) & 7 + 0 & -3 + 5 \\ 0 + 2 & -1 + 7 & -6 + (-7) \end{bmatrix} \quad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 9 & 7 & 2 \\ 2 & 6 & -13 \end{bmatrix} \quad \text{Simplify.}$$

b. $B - C$

$$B - C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix} - \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} -3 - 9 & 0 - 1 & 5 - (-5) \\ 2 - 0 & 7 - (-1) & -7 - 15 \end{bmatrix} \quad \text{Definition of matrix subtraction}$$

$$= \begin{bmatrix} -12 & -1 & 10 \\ 2 & 8 & -22 \end{bmatrix} \quad \text{Simplify.}$$

- You can multiply any matrix by a constant called a *scalar*. This is called **scalar multiplication**. To perform scalar multiplication, multiply each element by the scalar.

Example 2 **If $D = \begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix}$, find $2D$.**

$$2D = 2\begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 2(-4) & 2(6) & 2(-1) \\ 2(0) & 2(7) & 2(2) \\ 2(-3) & 2(-8) & 2(-4) \end{bmatrix} \quad \text{Definition of scalar multiplication}$$

$$= \begin{bmatrix} -8 & 12 & -2 \\ 0 & 14 & 4 \\ -6 & -16 & -8 \end{bmatrix} \quad \text{Simplify.}$$

- You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix. The product of two matrices is found by multiplying columns and rows. The entry in the first row and first column of *AB*, the resulting product, is found by multiplying corresponding elements in the first row of *A* and the first column of *B* and then adding.

Example 3 **Find *EF* if** $E = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix}$ **and** $F = \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$**.**

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$$

Multiply the numbers in the first row of *E* by the numbers in the first column of *F* and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & \\ & \end{bmatrix}$$

Multiply the numbers in the first row of *E* by the numbers in the second column of *F* and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ & \end{bmatrix}$$

Multiply the numbers in the second row of *E* by the numbers in the first column of *F* and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & \end{bmatrix}$$

Multiply the numbers in the second row of *E* by the numbers in the second column of *F* and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix}$$

Simplify the matrix.

$$\begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix} = \begin{bmatrix} -15 & 21 \\ 36 & -18 \end{bmatrix}$$

Exercises **If** $A = \begin{bmatrix} 10 & -9 \\ 4 & -3 \\ -1 & 11 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 2 & 8 \\ 7 & 6 \end{bmatrix}$, **and** $C = \begin{bmatrix} 8 & 0 \\ -2 & 2 \\ -10 & 6 \end{bmatrix}$, **find each sum, difference, or product.**

1. $A + B$ **2.** $B + C$ **3.** $A - C$ **4.** $C - B$

5. $3A$ **6.** $5B$ **7.** $-4C$ **8.** $\frac{1}{2}C$

9. $2A + C$ **10.** $A - 5C$ **11.** $\frac{1}{2}C + B$ **12.** $3A - 3B$

If $X = \begin{bmatrix} 2 & -8 \\ 10 & 4 \end{bmatrix}$, $Y = \begin{bmatrix} -1 & 0 \\ 6 & -5 \end{bmatrix}$, **and** $Z = \begin{bmatrix} 4 & -8 \\ -7 & 0 \end{bmatrix}$, **find each sum, difference, or product.**

13. $X + Z$ **14.** $Y + Z$ **15.** $X - Y$ **16.** $3Y$

17. $-6X$ **18.** $\frac{1}{2}X + Z$ **19.** $5Z - 2Y$ **20.** XY

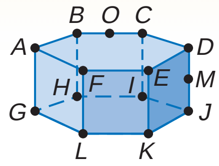
21. YZ **22.** XZ **23.** $\frac{1}{2}(XZ)$ **24.** $XY + 2Z$

Extra Practice

Lesson 1-1

(pages 6–12)

For Exercises 1–7, refer to the figure.



1. How many planes are shown in the figure?
2. Name three collinear points.
3. Name all planes that contain point *G*.
4. Name the intersection of plane *ABD* and plane *DJK*.
5. Name two planes that do not intersect.
6. Name a plane that contains $\overleftrightarrow{FK}$ and $\overleftrightarrow{EL}$.
7. Is the intersection of plane *ACD* and plane *EDJ* a point or a line? Explain.

Draw and label a figure for each relationship.

8. Line *a* intersects planes $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ at three distinct points.
9. Planes $\mathcal{X}$ and $\mathcal{Z}$ intersect in line *m*. Line *b* intersects the two planes in two distinct points.

Lesson 1-2

(pages 13–19)

Find the precision for each measurement. Explain its meaning.

1. 42 in.
2. 86 mm
3. 251 cm
4. 33.5 in.
5. $5\frac{1}{4}$ ft
6. 89 m

Find the value of the variable and *BC* if *B* is between *A* and *C*.

7. $AB = 4x$, $BC = 5x$; $AB = 16$
8. $AB = 17$, $BC = 3m$, $AC = 32$
9. $AB = 9a$, $BC = 12a$, $AC = 42$
10. $AB = 25$, $BC = 3b$, $AC = 7b + 13$
11. $AB = 5n + 5$, $BC = 2n$; $AC = 54$
12. $AB = 6c - 8$, $BC = 3c + 1$, $AC = 65$

Lesson 1-3

(pages 21–27)

Use the Pythagorean Theorem to find the distance between each pair of points.

1. $A(0, 0)$, $B(-3, 4)$
2. $C(-1, 2)$, $N(5, 10)$
3. $X(-6, -2)$, $Z(6, 3)$
4. $M(-5, -8)$, $O(3, 7)$
5. $T(-10, 2)$, $R(6, -10)$
6. $F(5, -6)$, $N(-5, 6)$

Use the Distance Formula to find the distance between each pair of points.

7. $D(0, 0)$, $M(8, -7)$
8. $X(-1, 1)$, $Y(1, -1)$
9. $Z(-4, 0)$, $A(-3, 7)$
10. $K(6, 6)$, $D(-3, -3)$
11. $T(-1, 3)$, $N(0, 2)$
12. $S(7, 2)$, $E(-6, 7)$

Find the coordinates of the midpoint of a segment having the given endpoints.

13. $A(0, 0)$, $D(-2, -8)$
14. $D(-4, -3)$, $E(2, 2)$
15. $K(-4, -5)$, $M(5, 4)$
16. $R(-10, 5)$, $S(8, 4)$
17. $B(2.8, -3.4)$, $Z(1.2, 5.6)$
18. $D(-6.2, 7)$, $K(3.4, -4.8)$

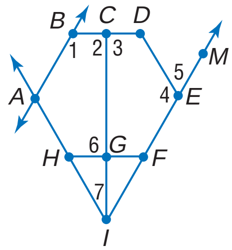
Find the coordinates of the missing endpoint given that *B* is the midpoint of $\overline{AC}$.

19. $C(0, 0)$, $B(5, -6)$
20. $C(-7, -4)$, $B(3, 5)$
21. $C(8, -4)$, $B(-10, 2)$
22. $C(6, 8)$, $B(-3, 5)$
23. $C(6, -8)$, $B(3, -4)$
24. $C(-2, -4)$, $B(0, 5)$

Lesson 1-4

(pages 29–36)

For Exercises 1–14, use the figure at the right.
Name the vertex of each angle.



1. $\angle 1$ **2.** $\angle 4$
3. $\angle 6$ **4.** $\angle 7$

Name the sides of each angle.

5. $\angle AIE$ **6.** $\angle 4$
7. $\angle 6$ **8.** $\angle AHF$

Write another name for each angle.

9. $\angle 3$ **10.** $\angle DEF$ **11.** $\angle 2$

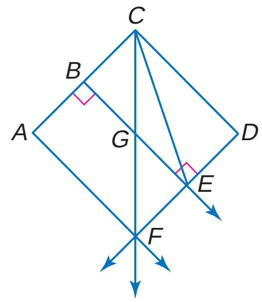
Measure each angle and classify it as *right*, *acute*, or *obtuse*.

12. $\angle ABC$ **13.** $\angle CGF$ **14.** $\angle HIF$

Lesson 1-5

(pages 37–43)

For Exercises 1–7, refer to the figure.



1. Name two acute vertical angles.
2. Name two obtuse vertical angles.
3. Name a pair of complementary adjacent angles.
4. Name a pair of supplementary adjacent angles.
5. Name a pair of equal supplementary adjacent angles.
6. If $m\angle BGC = 4x + 5$ and $m\angle FGE = 6x - 15$, find $m\angle BGF$.
7. If $m\angle BCG = 5a + 5$, $m\angle GCE = 3a - 4$, and $m\angle ECD = 4a - 7$, find the value of a so that $\overline{AC} \perp \overline{CD}$.

8. The measure of $\angle A$ is nine degrees less than the measure of $\angle B$. If $\angle A$ and $\angle B$ form a linear pair, what are their measures?

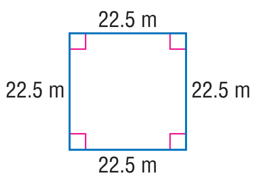
9. The measure of an angle's complement is 17 more than the measure of the angle. Find the measure of the angle and its complement.

Lesson 1-6

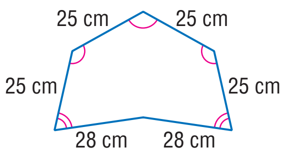
(pages 45–50)

Name each polygon by its number of sides. Classify it as *convex* or *concave* and *regular* or *irregular*. Then find the perimeter.

1.



2.



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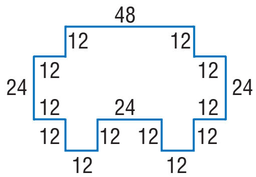


All measurements in inches.

Find the perimeter of each polygon.

4. triangle with vertices at $X(3, 3)$, $Y(-2, 1)$, and $Z(1, -3)$
5. pentagon with vertices at $P(-2, 3)$, $E(-5, 0)$, $N(-2, -4)$, $T(2, -1)$, and $A(2, 2)$
6. hexagon with vertices at $H(0, 4)$, $E(-3, 2)$, $X(-3, -2)$, $G(0, -5)$, $O(5, -2)$, and $N(5, 2)$

Extra Practice

Extra Practice

Lesson 2-1

(pages 62–66)

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture.

1. Lines *j* and *k* are parallel.
2. $A(-1, -7), B(4, -7), C(4, -3), D(-1, -3)$
3. $\overline{AB}$ bisects $\overline{CD}$ at *K*.
4. $\overrightarrow{RS}$ is an angle bisector of $\angle TSU$.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture.

5. **Given:** *EFG* is an equilateral triangle.
   Conjecture: $EF = FG$
6. **Given:** *r* is a rational number.
   Conjecture: *r* is a whole number.
7. **Given:** *n* is a whole number.
   Conjecture: *n* is a rational number.
8. **Given:** $\angle 1$ and $\angle 2$ are supplementary angles.
   Conjecture: $\angle 1$ and $\angle 2$ form a linear pair.

Lesson 2-2

(pages 67–74)

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value.

p: $(-3)^2 = 9$ **q: A robin is a fish.** **r: An acute angle measures less than 90°.**

1. p and q
2. p or q
3. p and r
4. p or r
5. $\sim p$ or q
6. p or $\sim r$
7. $q \wedge r$
8. $(p \wedge q) \vee r$
9. $\sim p \vee \sim r$

Copy and complete each truth table.

10.

p	q	$\sim q$	$p \vee \sim q$
T			
T			
F			
F			

11.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T			
T	F			
F	T			
F	F			

Lesson 2-3

(pages 75–80)

Identify the hypothesis and conclusion of each statement.

1. If no sides of a triangle are equal, then it is a scalene triangle.
2. If it rains today, you will be wearing your raincoat.
3. If $6 - x = 11$, then $x = -5$.
4. If you are in college, you are at least 18 years old.

Write each statement in if-then form.

5. The sum of the measures of two supplementary angles is 180.
6. A triangle with two equal sides is an isosceles triangle.
7. Two lines that never intersect are parallel lines.
8. A Saint Bernard is a dog.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

9. All triangles are polygons.
10. If two angles are congruent angles, then they have the same measure.
11. If two points lie on the same line, then they are collinear.
12. If $\overleftrightarrow{PQ}$ is a perpendicular bisector of $\overline{LM}$, then a right angle is formed.

Lesson 2-4

(pages 82–87)

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

1. (1) If it rains, then the field will be muddy.
(2) If the field is muddy, then the game will be cancelled.

2. (1) If you read a book, then you enjoy reading.
(2) If you are in the 10th grade, then you passed the 9th grade.

Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

3. (1) If it snows outside, you will wear your winter coat.
(2) It is snowing outside.
(3) You will wear your winter coat.

4. (1) Two complementary angles are both acute angles.
(2) $\angle 1$ and $\angle 2$ are acute angles.
(3) $\angle 1$ and $\angle 2$ are complementary angles.

Lesson 2-5

(pages 89–93)

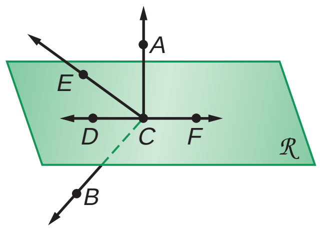
Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

1. $\overleftrightarrow{RS}$ is perpendicular to $\overleftrightarrow{PS}$.

2. Three points will lie on one line.

3. Points B and C are in plane $\mathcal{K}$. A line perpendicular to line BC is in plane $\mathcal{K}$.

For Exercises 4–7, use the figure at the right. In the figure, $\overleftrightarrow{EC}$ and $\overleftrightarrow{CD}$ are in plane $\mathcal{R}$, and F is on $\overleftrightarrow{CD}$. State the postulate that can be used to show each statement is true.



4. $\overleftrightarrow{DF}$ lies in plane $\mathcal{R}$.

5. E and C are collinear.

6. D, F, and E are coplanar.

7. E and F are collinear.

Lesson 2-6

(pages 94–100)

State the property that justifies each statement.

1. If $x - 5 = 6$, then $x = 11$.

2. If $AB = CD$ and $CD = EF$, then $AB = EF$.

3. If $a - b = r$, then $r = a - b$.

4. Copy and complete the following proof.

Given: $\frac{5x - 1}{8} = 3$

Prove: $x = 5$

Proof:

Statements	Reasons
a. ___?___	**a.** Given
b. ___?___	**b.** Multiplication Prop.
c. $5x - 1 = 24$	**c.** ___?___
d. $5x = 25$	**d.** ___?___
e. ___?___	**e.** Division Property

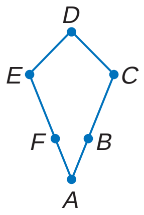
Lesson 2-7
(pages 101–106)

Justify each statement with a property of equality or a property of congruence.

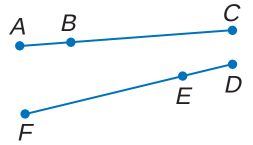
1. If $CD = OP$, then $CD + GH = OP + GH$.
2. If $\overline{MN} \cong \overline{PQ}$, then $\overline{PQ} \cong \overline{MN}$.
3. If $\overline{TU} \cong \overline{JK}$ and $\overline{JK} \cong \overline{DF}$, then $\overline{TU} \cong \overline{DF}$.
4. If $AB = 10$ and $CD = 10$, then $AB = CD$.
5. $\overline{XB} \cong \overline{XB}$
6. If $GH = RS$, then $GH - VW = RS - VW$.
7. If $EF = XY$, then $EF + KL = XY + KL$.
8. If $\overline{JK} \cong \overline{XY}$, and $\overline{XY} \cong \overline{LM}$, then $\overline{JK} \cong \overline{LM}$.

Write a two-column proof.

9. **Given:** $\overline{AF} \cong \overline{AB}$, $\overline{AF} \cong \overline{ED}$, $\overline{CD} \cong \overline{ED}$
   Prove: $\overline{AB} \cong \overline{CD}$



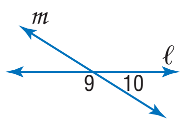
10. **Given:** $AC = DF$, $AB = DE$
    Prove: $BC = EF$



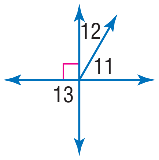
Lesson 2-8
(pages 107–114)

Find the measure of each numbered angle.

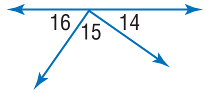
1. $m\angle 9 = 141 + x$
   $m\angle 10 = 25 + x$



2. $m\angle 11 = x + 40$
   $m\angle 12 = x + 10$
   $m\angle 13 = 3x + 30$



3. $m\angle 14 = x + 25$
   $m\angle 15 = 4x + 50$
   $m\angle 16 = x + 45$



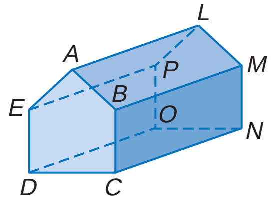
Determine whether the following statements are *always*, *sometimes*, or *never* true.

4. Two angles that are complementary are congruent.
5. Two angles that form a linear pair are complementary.
6. Two congruent angles are supplementary.
7. Perpendicular lines form four right angles.
8. Two right angles are supplementary.
9. Two lines intersect to form four right angles.

Lesson 3-1
(pages 126–131)

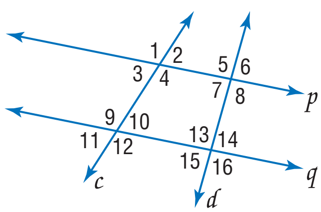
For Exercises 1–3, refer to the figure at the right.

1. Name all segments parallel to $\overline{AE}$.
2. Name all planes intersecting plane BCN.
3. Name all segments skew to $\overline{DC}$.



Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

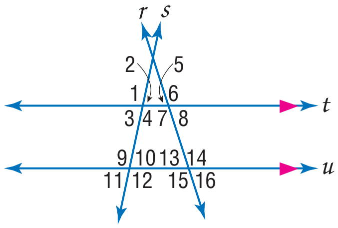
4. $\angle 2$ and $\angle 5$
5. $\angle 9$ and $\angle 13$
6. $\angle 12$ and $\angle 13$
7. $\angle 3$ and $\angle 6$



Lesson 3-2

(pages 133–138)

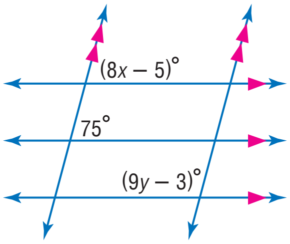
In the figure, $m\angle 5 = 72$ and $m\angle 9 = 102$. Find the measure of each angle.



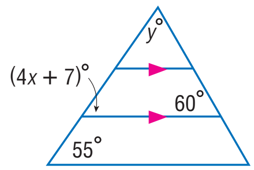
1. $m\angle 1$
2. $m\angle 13$
3. $m\angle 4$
4. $m\angle 10$
5. $m\angle 7$
6. $m\angle 16$

Find x and y in each figure.

7\.



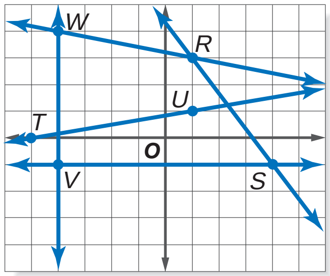
8\.



Lesson 3-3

(pages 139–144)

Find the slope of each line.



1. $\overleftrightarrow{RS}$
2. $\overleftrightarrow{TU}$
3. $\overleftrightarrow{WV}$
4. $\overleftrightarrow{WR}$
5. a line parallel to $\overleftrightarrow{TU}$
6. a line perpendicular to $\overleftrightarrow{WR}$
7. a line perpendicular to $\overleftrightarrow{WV}$

Determine whether $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$ are *parallel*, *perpendicular*, or *neither*.

8. $R(3, 5)$, $S(5, 6)$, $T(-2, 0)$, $U(4, 3)$
9. $R(5, 11)$, $S(2, 2)$, $T(-1, 0)$, $U(2, 1)$
10. $R(-1, 4)$, $S(-3, 7)$, $T(5, -1)$, $U(8, 1)$
11. $R(-2, 5)$, $S(-4, 1)$, $T(3, 3)$, $U(1, 5)$

Lesson 3-4

(pages 145–150)

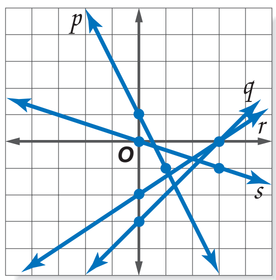
Write an equation in slope-intercept form of the line having the given slope and y-intercept.

1. $m = 1$, y-intercept: -5
2. $m = -\frac{1}{2}$, y-intercept: $\frac{1}{2}$
3. $m = 3$, $b = -\frac{1}{4}$

Write an equation in point-slope form of the line having the given slope that contains the given point.

4. $m = 3$, $(-2, 4)$
5. $m = -4$, $(0, 3)$
6. $m = \frac{2}{3}$, $(5, -7)$

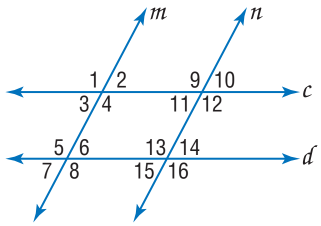
For Exercises 7–14, use the graph at the right. Write an equation in slope-intercept form for each line.



7. p
8. q
9. r
10. s
11. parallel to line q, contains $(2, -5)$
12. perpendicular to line r, contains $(0, 1)$
13. parallel to line s, contains $(-2, -2)$
14. perpendicular to line p, contains $(0, 0)$

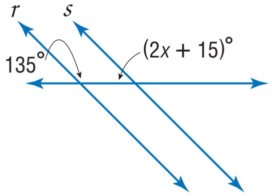
Lesson 3-5
(pages 151–157)

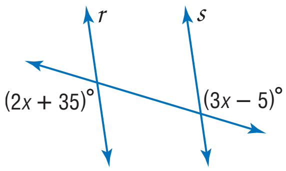
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

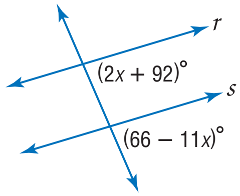


1. $\angle 9 \cong \angle 16$
2. $\angle 10 \cong \angle 16$
3. $\angle 12 \cong \angle 13$
4. $m\angle 12 + m\angle 14 = 180$

Find x so that $r \parallel s$.

5. 

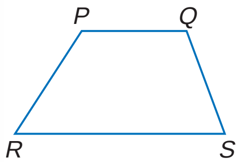
6. 

7. 

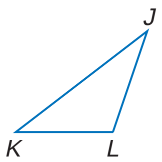
Lesson 3-6
(pages 159–164)

Copy each figure. Draw the segment that represents the distance indicated.

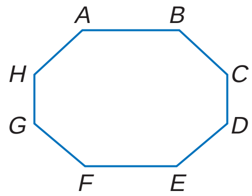
1. P to $\overline{RS}$



2. J to $\overline{KL}$



3. B to $\overline{FE}$



Find the distance between each pair of parallel lines.

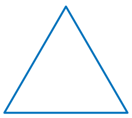
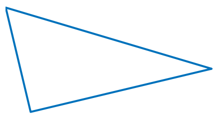
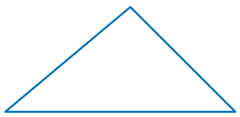
4. $y = \frac{2}{3}x - 2$
   $y = \frac{2}{3}x + \frac{1}{2}$
5. $y = 2x + 4$
   $y - 2x = -5$
6. $x + 4y = -6$
   $x + 4y = 4$

COORDINATE GEOMETRY **Construct a line perpendicular to ℓ through P. Then find the distance from P to ℓ.**

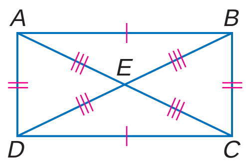
7. Line ℓ contains points (0, 4) and (−4, 0). Point P has coordinates (2, −1).
8. Line ℓ contains points (3, −2) and (0, 2). Point P has coordinates (−2.5, 3).

Lesson 4-1
(pages 178–183)

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

1. 
2. 
3. 

Identify the indicated type of triangles in the figure if $AB \cong CD$, $AD \cong BC$, $AE \cong BE \cong EC \cong ED$, and $m\angle BAD = m\angle ABC = m\angle BCD = m\angle ADC = 90$.



4. right
5. obtuse
6. acute
7. isosceles

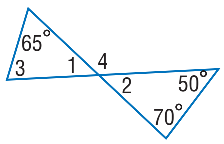
8. Find a and the measure of each side of equilateral triangle MNO if $MN = 5a$, $NO = 4a + 6$, and $MO = 7a - 12$.
9. Triangle TAC is an isosceles triangle with $\overline{TA} \cong \overline{AC}$. Find b, TA, AC, and TC if $TA = 3b + 1$, $AC = 4b - 11$, and $TC = 6b - 2$.

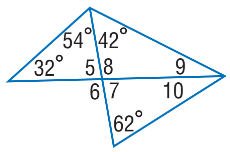
CONTENTS

Lesson 4-2 *(pages 185–191)*

Find the measure of each angle.

1. ∠1 **2.** ∠2
3. ∠3 **4.** ∠4
5. ∠5 **6.** ∠6
7. ∠7 **8.** ∠8
9. ∠9 **10.** ∠10

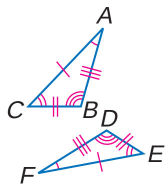




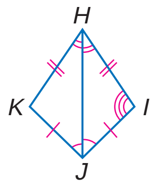
Lesson 4-3 *(pages 192–198)*

Identify the congruent triangles in each figure.

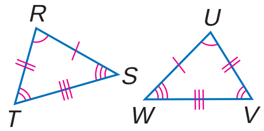
1.



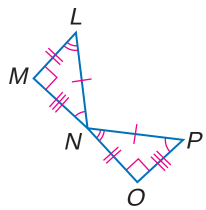
2.



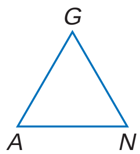
3.



4.



5. Write a two-column proof.
Given: △*ANG* ≅ △*NGA*
△*NGA* ≅ △*GAN*
Prove: △*AGN* is equilateral and equiangular.



Lesson 4-4 *(pages 200–206)*

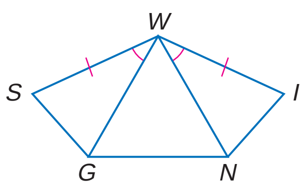
Determine whether △*RST* ≅ △*JKL* given the coordinates of the vertices. Explain.

1. *R*(−6, 2), *S*(−4, 4), *T*(−2, 2), *J*(6, −2), *K*(4, −4), *L*(2, −2)

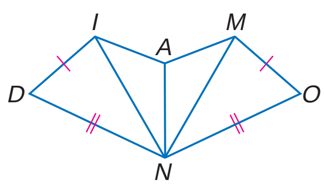
2. *R*(−6, 3), *S*(−4, 7), *T*(−2, 3), *J*(2, 3), *K*(5, 7), *L*(6, 3)

Write a two-column proof.

3. Given: △*GWN* is equilateral.
$\overline{WS} \cong \overline{WI}$
∠*SWG* ≅ ∠*IWN*
Prove: △*SWG* ≅ △*IWN*



4. Given: △*ANM* ≅ △*ANI*
$\overline{DI} \cong \overline{OM}$
$\overline{ND} \cong \overline{NO}$
Prove: △*DIN* ≅ △*OMN*

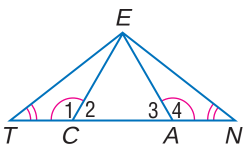


Lesson 4-5 *(pages 207–213)*

Write a paragraph proof.

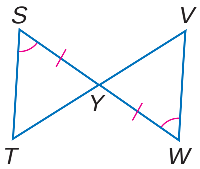
1. Given: $\triangle TEN$ is isosceles with base $\overline{TN}$.
$\angle 1 \cong \angle 4$, $\angle T \cong \angle N$

Prove: $\triangle TEC \cong \triangle NEA$



2. Given: $\angle S \cong \angle W$
$\overline{SY} \cong \overline{YW}$

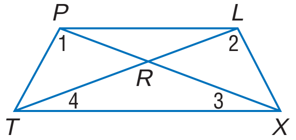
Prove: $\overline{ST} \cong \overline{WV}$



Write a flow proof.

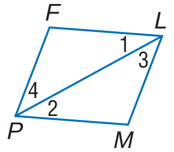
3. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\overline{PT} \cong \overline{LX}$



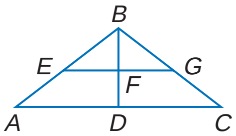
4. Given: $\overline{FP} \parallel \overline{ML}$, $\overline{FL} \parallel \overline{MP}$

Prove: $\overline{MP} \cong \overline{FL}$



Lesson 4-6 *(pages 216–221)*

Refer to the figure for Exercises 1–6.



1. If $\overline{AD} \cong \overline{BD}$, name two congruent angles.
2. If $\overline{BF} \cong \overline{FG}$, name two congruent angles.
3. If $\overline{BE} \cong \overline{BG}$, name two congruent angles.
4. If $\angle FBE \cong \angle FEB$, name two congruent segments.
5. If $\angle BCA \cong \angle BAC$, name two congruent segments.
6. If $\angle DBC \cong \angle BCD$, name two congruent segments.

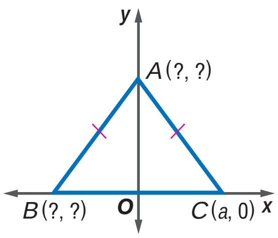
Lesson 4-7 *(pages 222–226)*

Position and label each triangle on the coordinate plane.

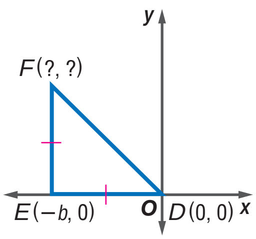
1. isosceles $\triangle ABC$ with base $\overline{BC}$ that is r units long
2. equilateral $\triangle XYZ$ with sides $4b$ units long
3. isosceles right $\triangle RST$ with hypotenuse $\overline{ST}$ and legs $(3 + a)$ units long
4. equilateral $\triangle CDE$ with base $\overline{DE}$ $\frac{1}{4}b$ units long.

Name the missing coordinates of each triangle.

5.



6.



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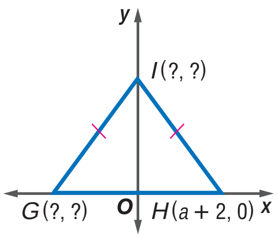
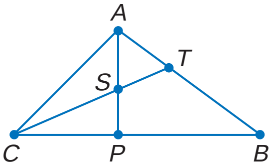
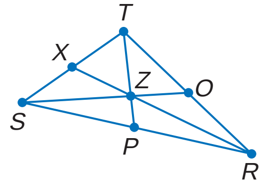


Lesson 5-1

(pages 238–246)

For Exercises 1–4, refer to the figures at the right.





1. Suppose $CP = 7x - 1$ and $PB = 6x + 3$. If S is the circumcenter of $\triangle ABC$, find x and CP.
2. Suppose $\angle ACT = 15a - 8$ and $\angle ACB = 74$. If S is the incenter of $\triangle ABC$, find a and $m\angle ACT$.
3. Suppose $TO = 7b + 5$, $OR = 13b - 10$, and $TR = 18b$. If Z is the centroid of $\triangle TRS$, find b and TR.
4. Suppose $XR = 19n - 14$ and $ZR = 10n + 4$. If Z is the centroid of $\triangle TRS$, find n and ZR.

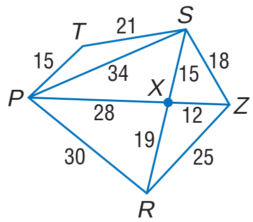
State whether each sentence is *always*, *sometimes*, or *never* true.

5. The circumcenter and incenter of a triangle are the same point.
6. The three altitudes of a triangle intersect at a point inside the triangle.
7. In an equilateral triangle, the circumcenter, incenter, and centroid are the same point.
8. The incenter is inside of a triangle.

Lesson 5-2

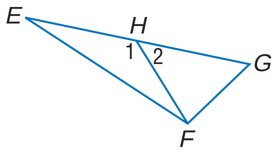
(pages 247–254)

Determine the relationship between the measures of the given angles.

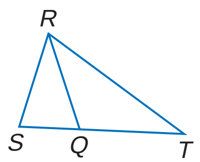


1. $\angle TPS$, $\angle TSP$
2. $\angle PRZ$, $\angle ZPR$
3. $\angle SPZ$, $\angle SZP$
4. $\angle SPR$, $\angle SRP$

5. **Given:** $FH > FG$
   Prove: $m\angle 1 > m\angle 2$



6. **Given:** $\overline{RQ}$ bisects $\angle SRT$.
   Prove: $m\angle SQR > m\angle SRQ$



Lesson 5-3

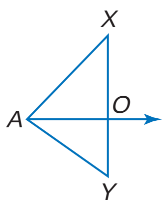
(pages 255–260)

State the assumption you would make to start an indirect proof of each statement.

1. $\angle ABC \cong \angle XYZ$
2. An angle bisector of an equilateral triangle is also a median.
3. $\overrightarrow{RS}$ bisects $\angle ARC$

Write an indirect proof.

4. **Given:** $\angle AOY \cong \angle AOX$
   $\overline{XO} \not\cong \overline{YO}$
   Prove: $\overrightarrow{AO}$ is not the angle bisector of $\angle XAY$.



5. **Given:** $\triangle RUN$
   Prove: There can be no more than one right angle in $\triangle RUN$.

Lesson 5-4
(pages 261–266)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*.

1. 2, 2, 6 **2.** 2, 3, 4 **3.** 6, 8, 10 **4.** 1, 1, 2

5. 15, 20, 30 **6.** 1, 3, 5 **7.** 2.5, 3.5, 6.5 **8.** 0.3, 0.4, 0.5

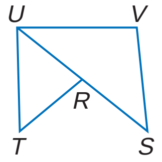
Find the range for the measure of the third side of a triangle given the measures of two sides.

9. 6 and 10 **10.** 2 and 5 **11.** 20 and 12 **12.** 8 and 8

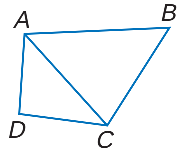
13. 18 and 36 **14.** 32 and 34 **15.** 2 and 29 **16.** 80 and 25

Write a two-column proof.

17. Given: $RS = RT$
Prove: $UV + VS > UT$



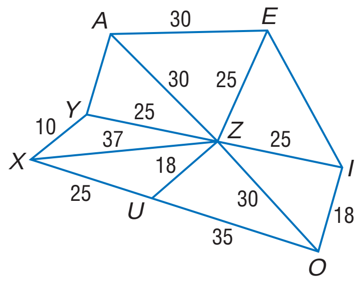
18. Given: quadrilateral $ABCD$
Prove: $AD + CD + AB > BC$



Lesson 5-5
(pages 267–273)

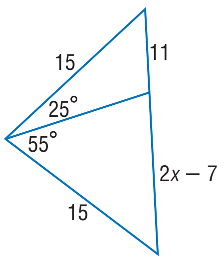
Write an inequality relating the given pair of angle or segment measures.

1. XZ, OZ
2. $m\angle ZIO$, $m\angle ZUX$
3. $m\angle AEZ$, $m\angle AZE$
4. IO, AE
5. $m\angle AZE$, $m\angle IZO$

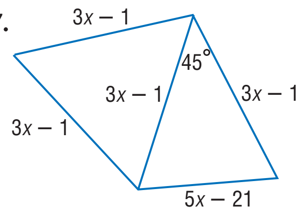


Write an inequality to describe the possible values of *x*.

6.



7.



Lesson 6-1
(pages 282–287)

1. **ARCHITECTURE** The ratio of the height of a model of a house to the actual house is 1:63. If the width of the model is 16 inches, find the width of the actual house in feet.

2. **CONSTRUCTION** A 64 inch long board is divided into lengths in the ratio 2:3. What are the two lengths into which the board is divided?

ALGEBRA **Solve each proportion.**

3. $\frac{x+4}{26} = -\frac{1}{3}$ **4.** $\frac{3x+1}{14} = \frac{5}{7}$ **5.** $\frac{x-3}{4} = \frac{x+1}{5}$ **6.** $\frac{2x+2}{2x-1} = \frac{1}{3}$

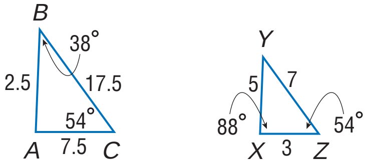
7. Find the measures of the sides of a triangle if the ratio of the measures of three sides of a triangle is 9:6:5, and its perimeter is 100 inches.

8. Find the measures of the angles in a triangle if the ratio of the measures of the three angles is 13:16:21.

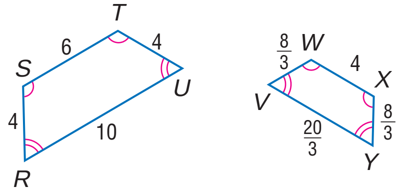
Lesson 6-2 (pages 289–297)

Determine whether each pair of figures is similar. Justify your answer.

1.



2.



For Exercises 3 and 4, use △*RST* with vertices *R*(3, 6), *S*(1, 2), and *T*(3, −1). Explain.

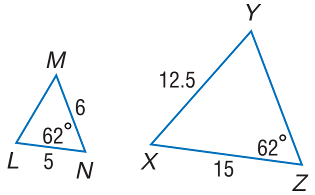
3. If the coordinates of each vertex are decreased by 3, describe the new figure. Is it similar to △*RST*?

4. If the coordinates of each vertex are multiplied by 0.5, describe the new figure. Is it similar to △*RST*?

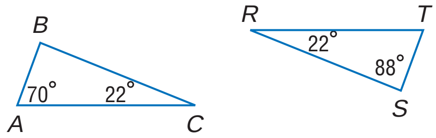
Lesson 6-3 (pages 298–306)

Determine whether each pair of triangles is similar. Justify your answer.

1.

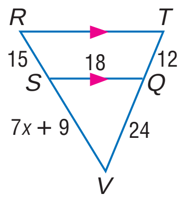


2.

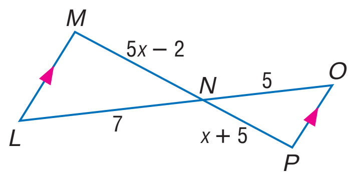


ALGEBRA **Identify the similar triangles. Find x and the measures of the indicated sides.**

3. *RT* and *SV*

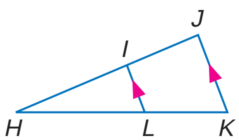


4. *PN* and *MN*

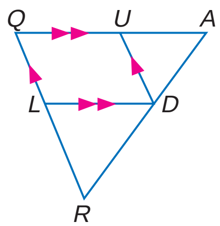


Lesson 6-4 (pages 307–315)

1. If $HI = 28$, $LH = 21$, and $LK = 8$, find *IJ*.



2. Find x, *AD*, *DR*, and *QR* if $AU = 15$, $QU = 25$, $AD = 3x + 6$, $DR = 8x - 2$, and $UD = 15$.

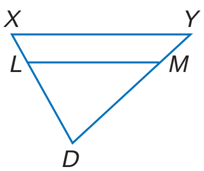


Find x so that $\overline{XY} \parallel \overline{LM}$.

3. $XL = 3$, $YM = 5$, $LD = 9$, $MD = x + 3$

4. $YM = 3$, $LD = 3x + 1$, $XL = 4$, $MD = x + 7$

5. $MD = 5x - 6$, $YM = 3$, $LD = 5x + 1$, $XL = 5$

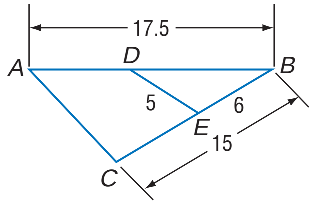


CONTENTS

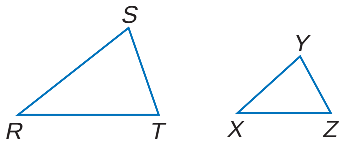
Lesson 6-5
(pages 316–323)

Find the perimeter of each triangle.

1. $\triangle ABC$ if $\triangle ABC \sim \triangle DBE$, $AB = 17.5$, $BC = 15$, $BE = 6$, and $DE = 5$



2. $\triangle RST$ if $\triangle RST \sim \triangle XYZ$, $RT = 12$, $XZ = 8$, and the perimeter of $\triangle XYZ = 22$

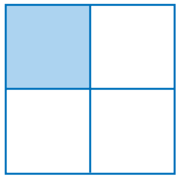


3. $\triangle LMN$ if $\triangle LMN \sim \triangle NXY$, $NX = 14$, $YX = 11$, $YN = 9$, and $LN = 27$

4. $\triangle GHI$ if $\triangle ABC \sim \triangle GHI$, $AB = 6$, $GH = 10$, and the perimeter of $\triangle ABC = 25$

Lesson 6-6
(pages 325–331)

Stage 1 of a fractal is shown drawn on grid paper. Stage 1 is made by dividing a square into 4 congruent squares and shading the top left-hand square.



1. Draw Stage 2 by repeating the Stage 1 process in each of the 3 remaining unshaded squares. How many shaded squares are at this stage?

2. Draw Stage 3 by repeating the Stage 1 process in each of the unshaded squares in Stage 2. How many shaded squares are at this stage?

Find the value of each expression. Then, use that value as the next x in the expression. Repeat the process and describe your observations.

3. $x^{\frac{1}{4}}$, where x initially equals 6
4. 4^x, where x initially equals 0.4
5. x^3, where x initially equals 0.5
6. 3^x, where x initially equals 10

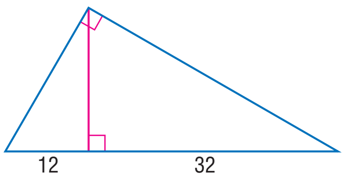
Lesson 7-1
(pages 342–348)

Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.

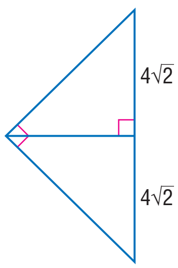
1. 8 and 12
2. 15 and 20
3. 1 and 2
4. 4 and 16
5. $3\sqrt{2}$ and $6\sqrt{2}$
6. $\frac{1}{2}$ and 10
7. $\frac{3}{8}$ and $\frac{1}{2}$
8. $\frac{\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$
9. $\frac{1}{10}$ and $\frac{7}{10}$

Find the altitude of each triangle.

10.



11.



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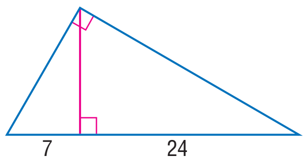


Lesson 7-2

(pages 350–356)

Determine whether $\triangle DEF$ is a right triangle for the given vertices. Explain.

1. $D(0, 1)$, $E(3, 2)$, $F(2, 3)$
2. $D(-2, 2)$, $E(3, -1)$, $F(-4, -3)$
3. $D(2, -1)$, $E(-2, -4)$, $F(-4, -1)$
4. $D(1, 2)$, $E(5, -2)$, $F(-2, -1)$

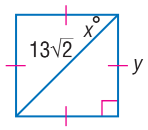
Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean triple.

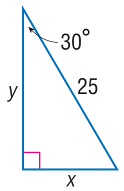
5. 1, 1, 2
6. 21, 28, 35
7. 3, 5, 7
8. 2, 5, 7
9. 24, 45, 51
10. $\frac{1}{3}, \frac{5}{3}, \frac{\sqrt{26}}{3}$
11. $\frac{6}{11}, \frac{8}{11}, \frac{10}{11}$
12. $\frac{1}{2}, \frac{1}{2}, 1$
13. $\frac{\sqrt{6}}{3}, \frac{\sqrt{10}}{5}, \frac{\sqrt{240}}{15}$

Lesson 7-3

(pages 357–363)

Find the measures of x and y.

1.


2.


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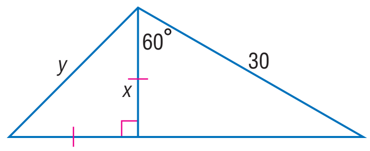

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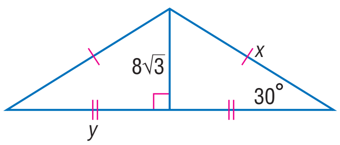

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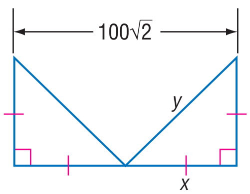

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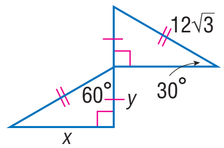
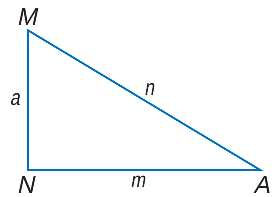


Lesson 7-4

(pages 364–370)

Use $\triangle MAN$ with right angle N to find $\sin M$, $\cos M$, $\tan M$, $\sin A$, $\cos A$, and $\tan A$. Express each ratio as a fraction, and as a decimal to the nearest hundredth.

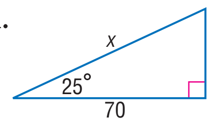


1. $m = 21, a = 28, n = 35$
2. $m = \sqrt{2}, a = \sqrt{3}, n = \sqrt{5}$
3. $m = \frac{\sqrt{2}}{2}, a = \frac{\sqrt{2}}{2}, n = 1$
4. $m = 3\sqrt{5}, a = 5\sqrt{3}, n = 2\sqrt{30}$

Find the measure of each angle to the nearest tenth of a degree.

5. $\cos A = 0.6293$
6. $\sin B = 0.5664$
7. $\tan C = 0.2665$
8. $\sin D = 0.9352$
9. $\tan M = 0.0808$
10. $\cos R = 0.1097$

Find x. Round to the nearest tenth.

11.


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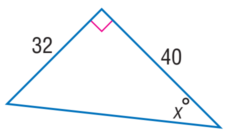

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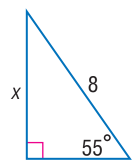
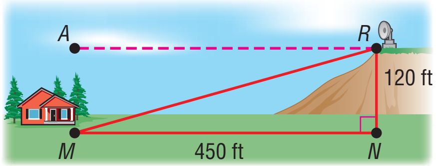


Extra Practice

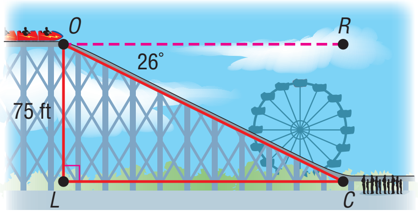
Lesson 7-5

(pages 371–376)

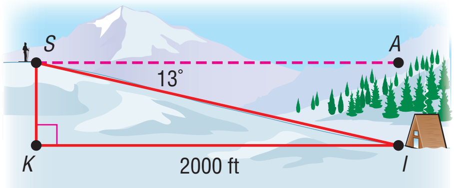
1. COMMUNICATIONS A house is located below a hill that has a satellite dish. If $MN = 450$ feet and $RN = 120$ feet, what is the measure of the angle of elevation to the top of the hill?



2. AMUSEMENT PARKS Mandy is at the top of the Mighty Screamer roller coaster. Her friend Bryn is at the bottom of the coaster waiting for the next ride. If the angle of depression from Mandy to Bryn is 26° and OL is 75 feet, what is the distance from L to C?



3. SKIING Mitchell is at the top of the Bridger Peak ski run. His brother Scott is looking up from the ski lodge at I. If the angle of elevation from Scott to Mitchell is 13° and the distance from K to I is 2000 ft, what is the length of the ski run SI?



Lesson 7-6

(pages 377–383)

Find each measure using the given measures from $\triangle ANG$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $m\angle N = 32$, $m\angle A = 47$, and $n = 15$, find a.

2. If $a = 10.5$, $m\angle N = 26$, $m\angle A = 75$, find n.

3. If $n = 18.6$, $a = 20.5$, $m\angle A = 65$, find $m\angle N$.

4. If $a = 57.8$, $n = 43.2$, $m\angle A = 33$, find $m\angle N$.

Solve each $\triangle AKX$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. $m\angle X = 62$, $a = 28.5$, $m\angle K = 33$

6. $k = 3.6$, $x = 2.1$, $m\angle X = 25$

7. $m\angle K = 35$, $m\angle A = 65$, $x = 50$

8. $m\angle A = 122$, $m\angle X = 15$, $a = 33.2$

Lesson 7-7

(pages 385–390)

In $\triangle CDE$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

1. $c = 100$, $d = 125$, $e = 150$; $m\angle E$

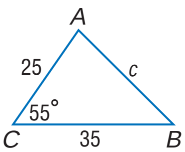
2. $c = 5$, $d = 6$, $e = 9$; $m\angle C$

3. $c = 1.2$, $d = 3.5$, $e = 4$; $m\angle D$

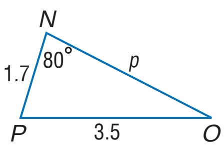
4. $c = 42.5$, $d = 50$, $e = 81.3$; $m\angle E$

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

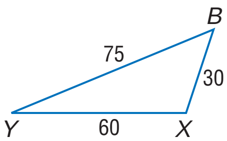
5.



6.



7.



Lesson 8-1

(pages 404–409)

Find the sum of the measures of the interior angles of each convex polygon.

1. 25-gon **2.** 30-gon **3.** 22-gon

4. 17-gon **5.** 5*a*-gon **6.** *b*-gon

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

7. 156 **8.** 168 **9.** 162

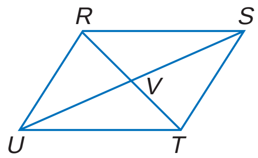
Find the measures of an interior angle and an exterior angle given the number of sides of a regular polygon. Round to the nearest tenth.

10. 15 **11.** 13 **12.** 42

Lesson 8-2

(pages 411–416)

Complete each statement about ▱*RSTU*. Justify your answer.

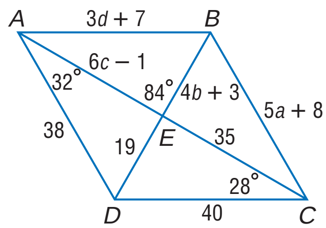


1. $\angle SRU \cong$ __?__ **2.** $\angle UTS$ is supplementary to __?__

3. $\overline{RU} \parallel$ __?__ **4.** $\overline{RU} \cong$ __?__

5. $\triangle RST \cong$ __?__ **6.** $\overline{SV} \cong$ __?__

ALGEBRA **Use ▱*ABCD* to find each measure or value.**



7. $m\angle BAE =$ __?__ **8.** $m\angle BCE =$ __?__

9. $m\angle BEC =$ __?__ **10.** $m\angle CED =$ __?__

11. $m\angle ABE =$ __?__ **12.** $m\angle EBC =$ __?__

13. $a =$ __?__ **14.** $b =$ __?__

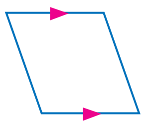
15. $c =$ __?__ **16.** $d =$ __?__

Lesson 8-3

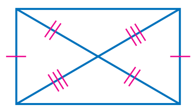
(pages 417–423)

Determine whether each quadrilateral is a parallelogram. Justify your answer.

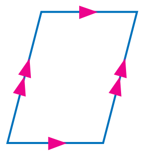
1.



2.

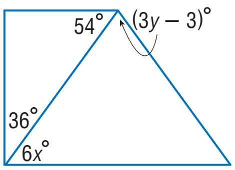


3.

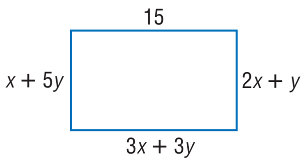


ALGEBRA **Find *x* and *y* so that each quadrilateral is a parallelogram.**

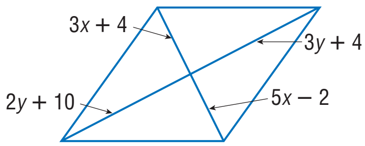
4.



5.



6.



Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

7. $L(-3, 2)$, $M(5, 2)$, $N(3, -6)$, $O(-5, -6)$; Slope Formula

8. $W(-5, 6)$, $X(2, 5)$, $Y(-3, -4)$, $Z(-8, -2)$; Distance Formula

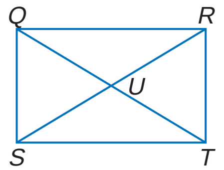
9. $Q(-5, 4)$, $R(0, 6)$, $S(3, -1)$, $T(-2, -3)$; Midpoint Formula

10. $G(-5, 0)$, $H(-13, 5)$, $I(-10, 9)$, $J(-2, 4)$; Distance and Slope Formulas

Lesson 8-4

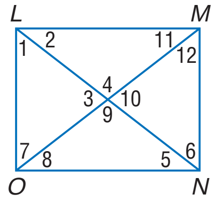
(pages 424–430)

ALGEBRA **Refer to rectangle *QRST*.**



1. If $QU = 2x + 3$ and $UT = 4x - 9$, find SU.
2. If $RU = 3x - 6$ and $UT = x + 9$, find RS.
3. If $QS = 3x + 40$ and $RT = 16 - 3x$, find QS.
4. If $m\angle STQ = 5x + 3$ and $m\angle RTQ = 3 - x$, find x.
5. If $m\angle SRQ = x^2 + 6$ and $m\angle RST = 36 - x$, find $m\angle SRT$.
6. If $m\angle TQR = x^2 + 16$ and $m\angle QTR = x + 32$, find $m\angle TQS$.

Find each measure in rectangle *LMNO* if $m\angle 5 = 38$.

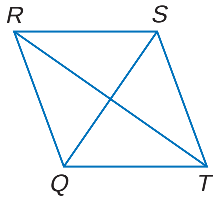


7. $m\angle 1$
8. $m\angle 2$
9. $m\angle 3$
10. $m\angle 4$
11. $m\angle 6$
12. $m\angle 7$
13. $m\angle 8$
14. $m\angle 9$
15. $m\angle 10$
16. $m\angle 11$
17. $m\angle 12$
18. $m\angle OLM$

Lesson 8-5

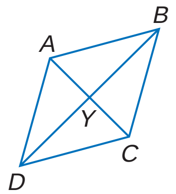
(pages 431–437)

In rhombus *QRST*, $m\angle QRS = m\angle TSR - 40$ and $TS = 15$.



1. Find $m\angle TSQ$.
2. Find $m\angle QRS$.
3. Find $m\angle SRT$.
4. Find QR.

ALGEBRA **Use rhombus *ABCD* with $AY = 6$, $DY = 3r + 3$, and $BY = \frac{10r - 4}{2}$.**



5. Find $m\angle ACB$.
6. Find $m\angle ABD$.
7. Find BY.
8. Find AC.

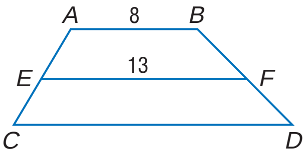
Lesson 8-6

(pages 439–445)

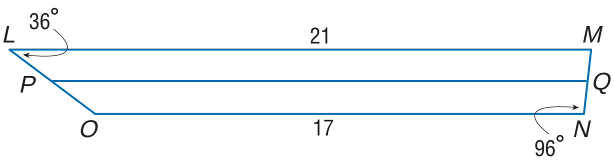
COORDINATE GEOMETRY **For each quadrilateral with the given vertices,**
a. verify that the quadrilateral is a trapezoid, and
b. determine whether the figure is an isosceles trapezoid.

1. $A(0, 9)$, $B(3, 4)$, $C(-5, 4)$, $D(-2, 9)$
2. $Q(1, 4)$, $R(4, 6)$, $S(10, 7)$, $T(1, 1)$
3. $L(1, 2)$, $M(4, -1)$, $N(3, -5)$, $O(-3, 1)$
4. $W(1, -2)$, $X(3, -1)$, $Y(7, -2)$, $Z(1, -5)$

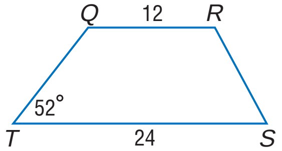
5. For trapezoid *ABDC*, *E* and *F* are midpoints of the legs. Find *CD*.



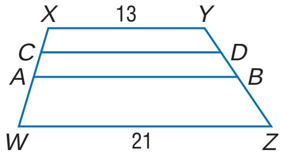
6. For trapezoid *LMNO*, *P* and *Q* are midpoints of the legs. Find *PQ*, $m\angle M$, and $m\angle O$.



7. For isosceles trapezoid *QRST*, find the length of the median, $m\angle S$, and $m\angle R$.



8. For trapezoid *XYZW*, *A* and *B* are midpoints of the legs. For trapezoid *XYBA*, *C* and *D* are midpoints of the legs. Find *CD*.

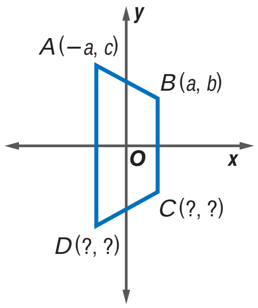


Lesson 8-7

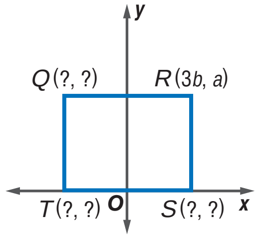
(pages 447–451)

Name the missing coordinates for each quadrilateral.

1. isosceles trapezoid *ABCD*



2. rectangle *QRST*



Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following.

3. The diagonals of a square are congruent.

4. Quadrilateral *EFGH* with vertices $E(0, 0)$, $F(a\sqrt{2}, a\sqrt{2})$, $G(2a + a\sqrt{2}, a\sqrt{2})$, and $H(2a, 0)$ is a rhombus.

Lesson 9-1

(pages 463–469)

COORDINATE GEOMETRY **Graph each figure and the image of the given reflection.**

1. $\triangle ABN$ with vertices $A(2, 2)$, $B(3, -2)$, and $N(-3, -1)$ in the x-axis

2. rectangle *BARN* with vertices $B(3, 3)$, $A(3, -4)$, $R(-1, -4)$, and $N(-1, 3)$ in the line $y = x$

3. trapezoid *ZOID* with vertices $Z(2, 3)$, $O(2, -4)$, $I(-3, -3)$, and $D(-3, 1)$ in the origin

4. $\triangle PQR$ with vertices $P(-2, 1)$, $Q(2, -2)$, and $R(-3, -4)$ in the y-axis

5. square *BDFH* with vertices $B(-4, 4)$, $D(-1, 4)$, $F(-1, 1)$, and $H(-4, 1)$ in the origin

6. quadrilateral *QUAD* with vertices $Q(1, 3)$, $U(3, 1)$, $A(-1, 0)$, and $D(-3, 4)$ in the line $y = -1$

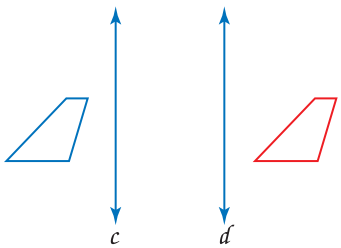
7. $\triangle CAB$ with vertices $C(0, 4)$, $A(1, -3)$, and $B(-4, 0)$ in the line $x = -2$

Lesson 9-2

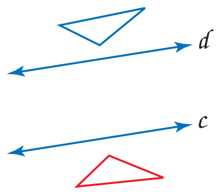
(pages 470–475)

In each figure, $c \parallel d$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer.

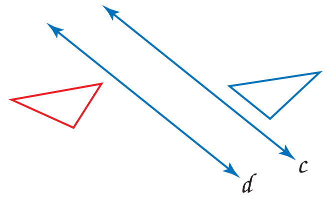
1.



2.



3.



COORDINATE GEOMETRY **Graph each figure and its image under the given translation.**

4. $\overline{LM}$ with endpoints $L(2, 3)$ and $M(-4, 1)$ under the translation $(x, y) \rightarrow (x + 2, y + 1)$

5. $\triangle DEF$ with vertices $D(1, 2)$, $E(-2, 1)$, and $F(-3, -1)$ under the translation $(x, y) \rightarrow (x - 1, y - 3)$

6. quadrilateral *WXYZ* with vertices $W(1, 1)$, $X(-2, 3)$, $Y(-3, -2)$, and $Z(2, -2)$ under the translation $(x, y) \rightarrow (x + 1, y - 1)$

7. pentagon *ABCDE* with vertices $A(1, 3)$, $B(-1, 1)$, $C(-1, -2)$, $D(3, -2)$, and $E(3, 1)$ under the translation $(x, y) \rightarrow (x - 2, y + 3)$

8. $\triangle RST$ with vertices $R(-4, 3)$, $S(-2, -3)$, and $T(2, -1)$ under the translation $(x, y) \rightarrow (x + 3, y - 2)$

Lesson 9-3

(pages 476–482)

COORDINATE GEOMETRY **Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates.**

1. $\triangle KLM$ with vertices $K(4, 2)$, $L(1, 3)$, and $M(2, 1)$ counterclockwise about the point $P(1, -1)$
2. $\triangle FGH$ with vertices $F(-3, -3)$, $G(2, -4)$, and $H(-1, -1)$ clockwise about the point $P(0, 0)$

COORDINATE GEOMETRY **Draw the rotational image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.**

3. $\triangle HIJ$ with vertices $H(2, 2)$, $I(-2, 1)$, and $J(-1, -2)$, reflected in the x-axis and then in the y-axis
4. $\triangle NOP$ with vertices $N(3, 1)$, $O(5, -3)$, and $P(2, -3)$, reflected in the y-axis and then in the line $y = x$
5. $\triangle QUA$ with vertices $Q(0, 4)$, $U(-3, 2)$, and $A(1, 1)$, reflected in the x-axis and then in the line $y = x$
6. $\triangle AEO$ with vertices $A(-5, 3)$, $E(-4, 1)$, and $O(-1, 2)$, reflected in the line $y = -x$ and then in the y-axis

Lesson 9-4

(pages 483–488)

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

1. regular hexagons and squares
2. squares and regular pentagons
3. regular hexagons and regular octagons

Determine whether each statement is *always, sometimes,* or *never* true.

4. Any right isosceles triangle can be used to form a uniform tessellation.
5. A semi-regular tessellation is uniform.
6. A polygon that is not regular can tessellate the plane.
7. If the measure of one interior angle of a regular polygon is greater than 120°, it cannot tessellate the plane.

Lesson 9-5

(pages 490–497)

Find the measure of the dilation image or the preimage of $\overline{OM}$ with the given scale factor.

1. $OM = 1, r = -2$
2. $OM = 3, r = \frac{1}{3}$
3. $O'M' = \frac{3}{4}, r = 3$
4. $OM = \frac{7}{8}, r = -\frac{5}{7}$
5. $O'M' = 4, r = -\frac{2}{3}$
6. $O'M' = 4.5, r = -1.5$

COORDINATE GEOMETRY **Find the image of each polygon, given the vertices, after a dilation centered at the origin with scale factor $r = 3$. Then graph a dilation with $r = \frac{1}{3}$.**

7. $T(1, 1)$, $R(-1, 2)$, $I(-2, 0)$
8. $E(2, 1)$, $I(3, -3)$, $O(-1, -2)$
9. $A(0, -1)$, $B(-1, 1)$, $C(0, 2)$, $D(1, 1)$
10. $B(1, 0)$, $D(2, 0)$, $F(3, -2)$, $H(0, -2)$

Lesson 9-6

(pages 498–505)

Find the magnitude and direction of $\overrightarrow{XY}$ for the given coordinates.

1. $X(1, 1)$, $Y(-2, 3)$
2. $X(-1, -1)$, $Y(2, 2)$
3. $X(-5, 4)$, $Y(-2, -3)$
4. $X(2, 1)$, $Y(-4, -4)$
5. $X(-2, -1)$, $Y(2, -2)$
6. $X(3, -1)$, $Y(-3, 1)$

Graph the image of each figure under a translation by the given vector.

7. $\triangle HIJ$ with vertices $H(2, 3)$, $I(-4, 2)$, $J(-1, 1)$; $\vec{\mathbf{a}} = \langle 1, 3 \rangle$
8. quadrilateral $RSTW$ with vertices $R(4, 0)$, $S(0, 1)$, $T(-2, -2)$, $W(3, -1)$; $\vec{\mathbf{x}} = \langle -3, 4 \rangle$
9. pentagon $AEIOU$ with vertices $A(-1, 3)$, $E(2, 3)$, $I(2, 0)$, $O(-1, -2)$, $U(-3, 0)$; $\vec{\mathbf{b}} = \langle -2, -1 \rangle$

Find the magnitude and direction of each resultant for the given vectors.

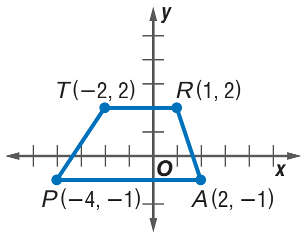
10. $\vec{\mathbf{c}} = \langle 2, 3 \rangle$, $\vec{\mathbf{d}} = \langle 3, 4 \rangle$
11. $\vec{\mathbf{a}} = \langle 1, 3 \rangle$, $\vec{\mathbf{b}} = \langle -4, 3 \rangle$
12. $\vec{\mathbf{x}} = \langle 1, 2 \rangle$, $\vec{\mathbf{y}} = \langle 4, -6 \rangle$
13. $\vec{\mathbf{s}} = \langle 2, 5 \rangle$, $\vec{\mathbf{t}} = \langle -6, -8 \rangle$
14. $\vec{\mathbf{m}} = \langle 2, -3 \rangle$, $\vec{\mathbf{n}} = \langle -2, 3 \rangle$
15. $\vec{\mathbf{u}} = \langle -7, 2 \rangle$, $\vec{\mathbf{v}} = \langle 4, 1 \rangle$

Lesson 9-7

(pages 506–511)

Find the coordinates of the image under the stated transformation.

1. reflection in the x-axis
2. rotation 90° clockwise about the origin
3. translation $(x, y) \rightarrow (x - 4, y + 3)$
4. dilation by scale factor -4



Use a matrix to find the coordinates of the vertices of the image of each figure after the stated transformation.

5. $\triangle DEF$ with $D(2, 4)$, $E(-2, -4)$, and $F(4, -6)$; dilation by a scale factor of 2.5
6. $\triangle RST$ with $R(3, 4)$, $S(-6, -2)$, and $T(5, -3)$; reflection in the x-axis
7. quadrilateral $CDEF$ with $C(1, 1)$, $D(-2, 5)$, $E(-2, 0)$, and $F(-1, -2)$; rotation of 90° counterclockwise
8. quadrilateral $WXYZ$ with $W(0, 4)$, $X(-5, 0)$, $Y(0, -3)$, and $Z(5, -2)$; translation $(x, y) \rightarrow (x + 1, y - 4)$
9. quadrilateral $JKLM$ with $J(-6, -2)$, $K(-2, -8)$, $L(4, -4)$, and $M(6, 6)$; dilation by a scale factor of $-\frac{1}{2}$
10. pentagon $ABCDE$ with $A(2, 2)$, $B(0, 4)$, $C(-3, 2)$, $D(-3, -4)$, and $E(2, -4)$; reflection in the line $y = x$

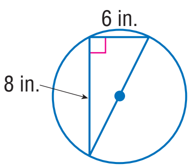
Lesson 10-1

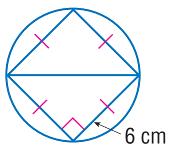
(pages 522–528)

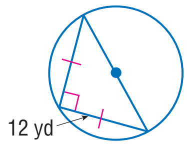
The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

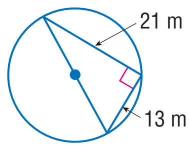
1. $r = 18$ in., $d =$ __?__, $C =$ __?__
2. $d = 34.2$ ft, $r =$ __?__, $C =$ __?__
3. $C = 12\pi$ m, $r =$ __?__, $d =$ __?__
4. $C = 84.8$ mi, $r =$ __?__, $d =$ __?__
5. $d = 8.7$ cm, $r =$ __?__, $C =$ __?__
6. $r = 3b$ in., $d =$ __?__, $C =$ __?__

Find the exact circumference of each circle.

7.


8.


9.


10.


Extra Practice

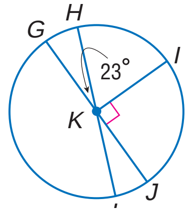
CONTENTS

Lesson 10-2

(pages 529–535)

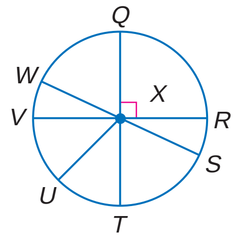
Find each measure.

1. $m\angle GKI$
2. $m\angle LKJ$
3. $m\angle LKI$
4. $m\angle LKG$
5. $m\angle HKI$
6. $m\angle HKJ$



In $\odot X$, $\overline{WS}$, $\overline{VR}$, and $\overline{QT}$ are diameters, $m\angle WXV = 25$ and $m\angle VXU = 45$. Find each measure.

7. $m\overset{\frown}{QR}$
8. $m\overset{\frown}{QW}$
9. $m\overset{\frown}{TU}$
10. $m\overset{\frown}{WRV}$
11. $m\overset{\frown}{SV}$
12. $m\overset{\frown}{TRW}$

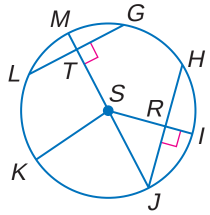


Lesson 10-3

(pages 536–543)

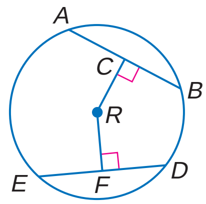
In $\odot S$, $HJ = 22$, $LG = 18$, $m\overset{\frown}{IJ} = 35$, and $m\overset{\frown}{LM} = 30$. Find each measure.

1. HR
2. RJ
3. LT
4. TG
5. $m\overset{\frown}{HJ}$
6. $m\overset{\frown}{LG}$
7. $m\overset{\frown}{MG}$
8. $m\overset{\frown}{HI}$



In $\odot R$, $CR = RF$, and $ED = 30$. Find each measure.

9. AB
10. EF
11. DF
12. BC

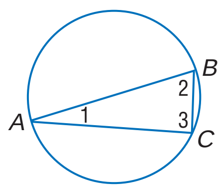


Lesson 10-4

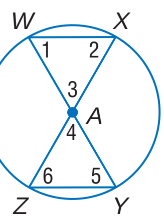
(pages 544–551)

Find the measure of each numbered angle for each figure.

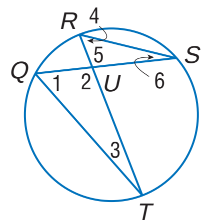
1. $m\overset{\frown}{AB} = 176$, and $m\overset{\frown}{BC} = 42$



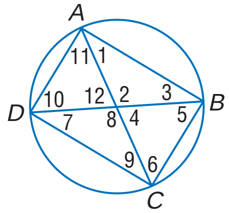
2. $\overline{WX} \cong \overline{ZY}$, and $m\overset{\frown}{ZW} = 120$



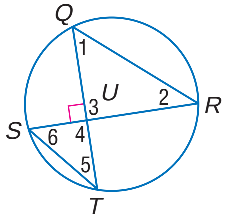
3. $m\overset{\frown}{QR} = 40$, and $m\overset{\frown}{TS} = 110$



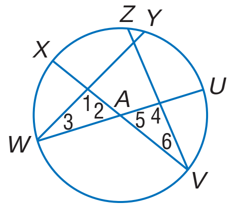
4. $\square ABCD$ is a rectangle, and $m\overset{\frown}{BC} = 70$.



5. $m\overset{\frown}{TR} = 100$, and $\overline{SR} \perp \overline{QT}$



6. $m\overset{\frown}{UY} = m\overset{\frown}{XZ} = 56$ and $m\overset{\frown}{UV} = m\overset{\frown}{XW} = 56$



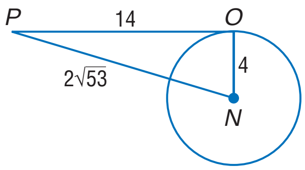
7. Rhombus $ABCD$ is inscribed in a circle. What can you conclude about $\overline{BC}$?
8. Triangle RST is inscribed in a circle. If the measure of $\overset{\frown}{RS}$ is 170, what is the measure of $\angle T$?

Lesson 10-5

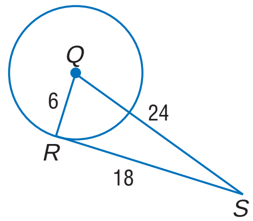
(pages 552–558)

Determine whether each segment is tangent to the given circle.

1.

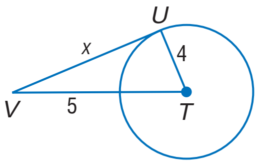


2.

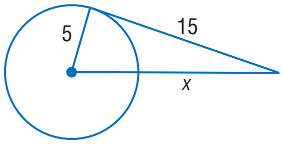


Find x. Assume that segments that appear to be tangent are tangent.

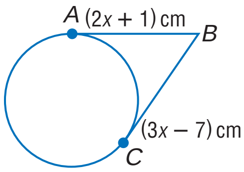
3.



4.



5.

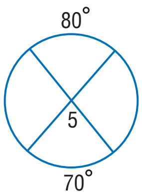


Lesson 10-6

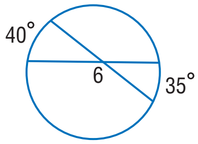
(pages 561–568)

Find each measure.

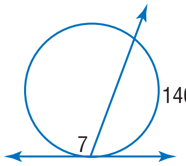
1. $m\angle 5$



2. $m\angle 6$

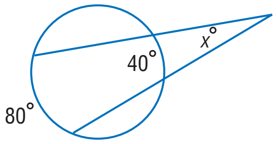


3. $m\angle 7$

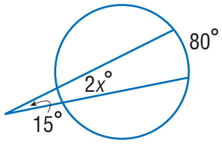


Find x. Assume that any segment that appears to be tangent is tangent.

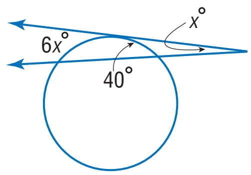
4.



5.



6.

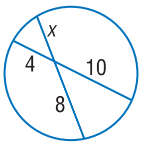


Lesson 10-7

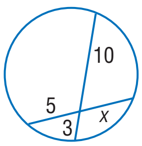
(pages 569–574)

Find x to the nearest tenth. Assume that segments that appear to be tangent are tangent.

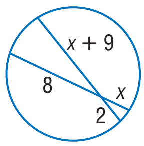
1.



2.

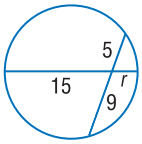


3.

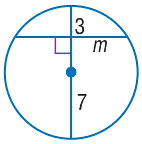


Find each variable to the nearest tenth.

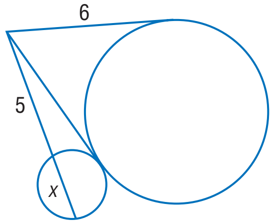
4.



5.



6.



CONTENTS

Lesson 10-8 *(pages 575–580)*

Write an equation for each circle.

1. center at $(1, -2)$, $r = 2$
2. center at origin, $r = 4$
3. center at $(-3, -4)$, $r = \sqrt{11}$
4. center at $(3, -1)$, $d = 6$
5. center at $(6, 12)$, $r = 7$
6. center at $(4, 0)$, $d = 8$
7. center at $(6, -6)$, $d = 22$
8. center at $(-5, 1)$, $d = 2$

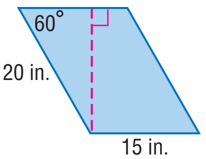
Graph each equation.

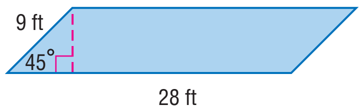
9. $x^2 + y^2 = 25$
10. $x^2 + y^2 - 3 = 1$
11. $(x - 3)^2 + (y + 1)^2 = 9$
12. $(x - 1)^2 + (y - 4)^2 = 1$

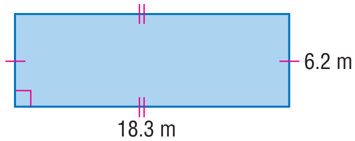
13. Find the radius of a circle whose equation is $(x + 3)^2 + (y - 1)^2 = r^2$ and contains $(-2, 1)$.
14. Find the radius of a circle whose equation is $(x - 4)^2 + (y - 3)^2 = r^2$ and contains $(8, 3)$.

Lesson 11-1 *(pages 595–600)*

Find the area and perimeter of each parallelogram. Round to the nearest tenth if necessary.

1. 

2. 

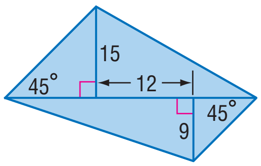
3. 

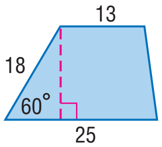
COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

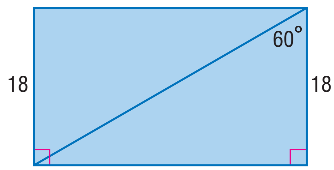
4. $Q(-3, 3)$, $R(-1, 3)$, $S(-1, 1)$, $T(-3, 1)$
5. $A(-7, -6)$, $B(-2, -6)$, $C(-2, -3)$, $D(-7, -3)$
6. $L(5, 3)$, $M(8, 3)$, $N(9, 7)$, $O(6, 7)$
7. $W(-1, -2)$, $X(-1, 1)$, $Y(2, 1)$, $Z(2, -2)$

Lesson 11-2 *(pages 601–609)*

Find the area of each quadrilateral.

1. 

2. 

3. 

COORDINATE GEOMETRY **Find the area of trapezoid *ABCD* given the coordinates of the vertices.**

4. $A(1, 1)$, $B(2, 3)$, $C(4, 3)$, $D(7, 1)$
5. $A(-2, 2)$, $B(2, 2)$, $C(7, -3)$, $D(-4, -3)$
6. $A(1, -1)$, $B(4, -1)$, $C(8, 5)$, $D(1, 5)$
7. $A(-2, 2)$, $B(4, 2)$, $C(3, -2)$, $D(1, -2)$

COORDINATE GEOMETRY **Find the area of rhombus *LMNO* given the coordinates of the vertices.**

8. $L(-3, 0)$, $M(1, -2)$, $N(-3, -4)$, $O(-7, -2)$
9. $L(-3, -2)$, $M(-4, 2)$, $N(-3, 6)$, $O(-2, 2)$
10. $L(-1, -4)$, $M(3, 4)$, $N(-1, 12)$, $O(-5, 4)$
11. $L(-2, -2)$, $M(4, 4)$, $N(10, -2)$, $O(4, -8)$

Lesson 11-3

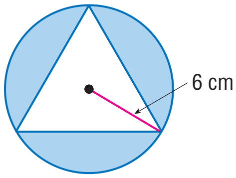
(pages 610–616)

Find the area of each regular polygon. Round to the nearest tenth.

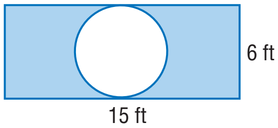
1. a square with perimeter 54 feet
2. a triangle with side length 9 inches
3. an octagon with side length 6 feet
4. a decagon with apothem length of 22 centimeters

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

5.



6.



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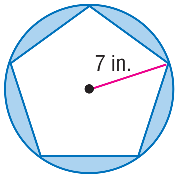
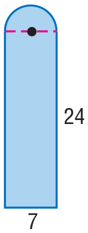


Lesson 11-4

(pages 617–621)

Find the area of each figure. Round to the nearest tenth if necessary.

1.



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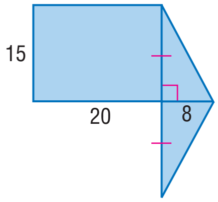
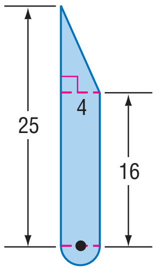


3.



COORDINATE GEOMETRY **The vertices of an irregular figure are given. Find the area of each figure.**

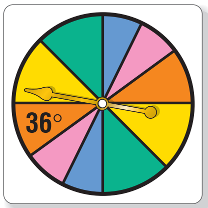
4. $R(0, 5)$, $S(3, 3)$, $T(3, 0)$
5. $A(-5, -3)$, $B(-3, 0)$, $C(2, -1)$, $D(2, -3)$
6. $L(-1, 4)$, $M(3, 2)$, $N(3, -1)$, $O(-1, -2)$, $P(-3, 1)$

Lesson 11-5

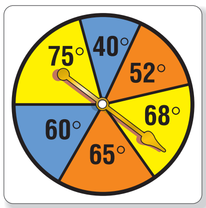
(pages 622–627)

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 20 inches.

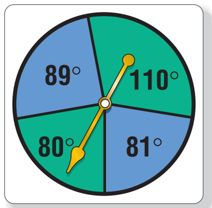
1. orange



2. blue

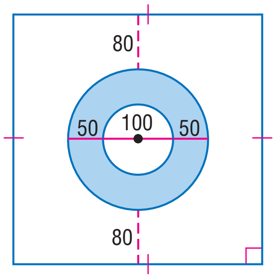


3. green

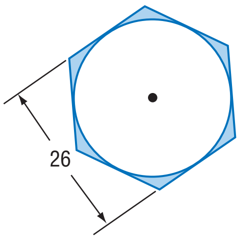


Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume all inscribed polygons are regular.

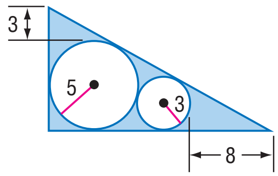
4.



5.



6.

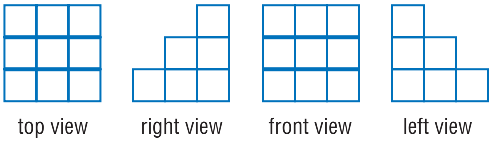


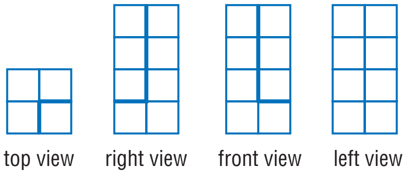
Extra Practice

Lesson 12-1

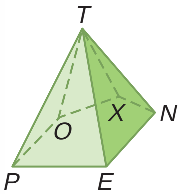
(pages 636–642)

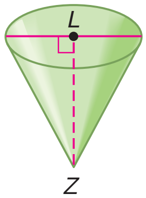
Draw the back view and corner view of a figure given its orthogonal drawing.

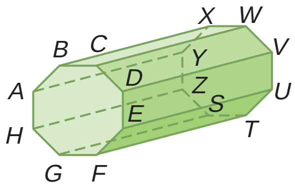
1. 

2. 

Identify each solid. Name the bases, faces, edges, and vertices.

3. 

4. 

5. 

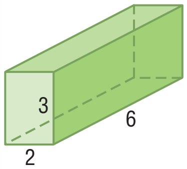
Lesson 12-2

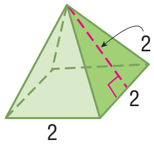
(pages 643–648)

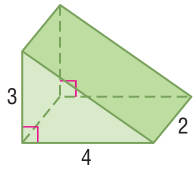
Sketch each solid using isometric dot paper.

1. rectangular prism 2 units high, 3 units long, and 2 units wide
2. rectangular prism 1 unit high, 2 units long, and 3 units wide
3. triangular prism 3 units high with bases that are right triangles with legs 3 units and 4 units long
4. triangular prism 5 units high with bases that are right triangles with legs 4 units and 6 units long

For each solid, draw a net and find the surface area. Round to the nearest tenth if necessary.

5. 

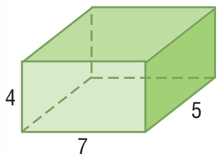
6. 

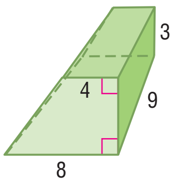
7. 

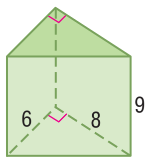
Lesson 12-3

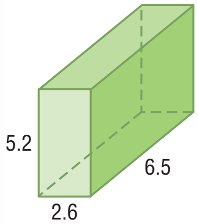
(pages 649–654)

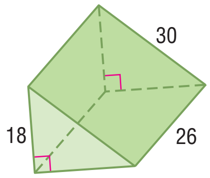
Find the lateral area and the surface area of each prism. Round to the nearest tenth if necessary.

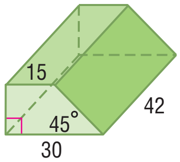
1. 

2. 

3. 

4. 

5. 

6. 

7. The surface area of a right triangular prism is 228 square inches. The base has legs measuring 6 inches and 8 inches. Find the height of the prism.

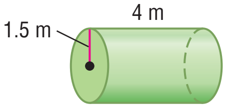
8. The surface area of a right triangular prism with height 18 inches is 1,380 square inches. The base has a leg measuring 15 inches and a hypotenuse of length 25 inches. Find the length of the other leg of the base.

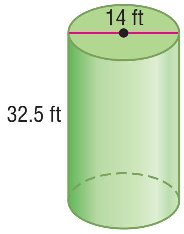
Lesson 12-4 (pages 655–659)

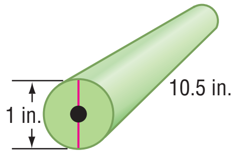
Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

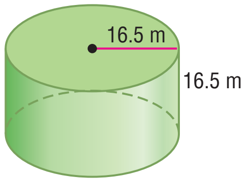
1. $r = 2$ ft, $h = 3.5$ ft
2. $d = 15$ in., $h = 20$ in.
3. $r = 3.7$ m, $h = 6.2$ m
4. $d = 19$ mm, $h = 32$ mm

Find the surface area of each cylinder. Round to the nearest tenth.

5. 

6. 

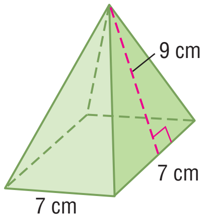
7. 

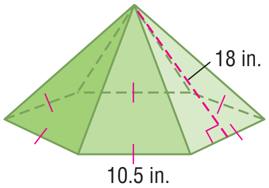
8. 

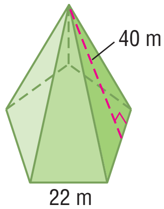
Extra Practice

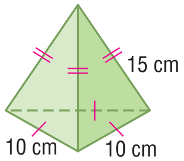
Lesson 12-5 (pages 660–665)

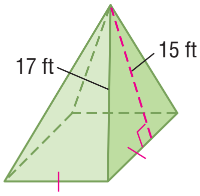
Find the surface area of each regular pyramid. Round to the nearest tenth.

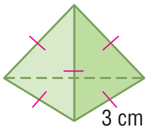
1. 

2. 

3. 

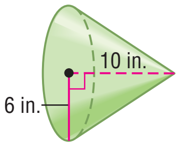
4. 

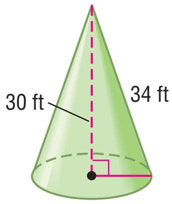
5. 

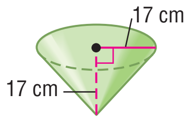
6. 

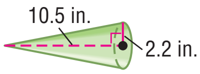
Lesson 12-6 (pages 666–670)

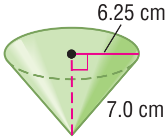
Find the surface area of each cone. Round to the nearest tenth.

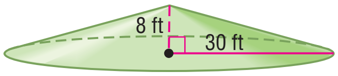
1. 

2. 

3. 

4. 

5. 

6. 

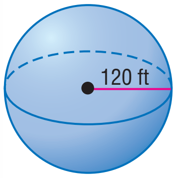
7. Find the surface area of a cone if the height is 28 inches and the slant height is 40 inches.

8. Find the surface area of a cone if the height is 7.5 centimeters and the radius is 2.5 centimeters.

Lesson 12-7

(pages 671–676)

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1.


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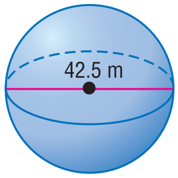

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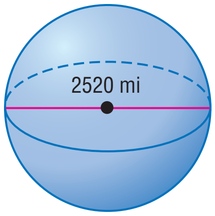

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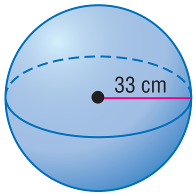


5. a hemisphere with the circumference of a great circle 14.1 cm

6. a sphere with the circumference of a great circle 50.3 in.

7. a sphere with the area of a great circle 98.5 m^2

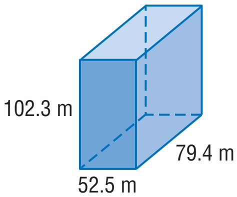
8. a hemisphere with the circumference of a great circle 3.1 in.

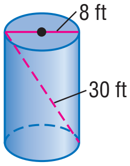
9. a hemisphere with the area of a great circle 31,415.9 ft^2

Lesson 13-1

(pages 688–694)

Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1.


2.


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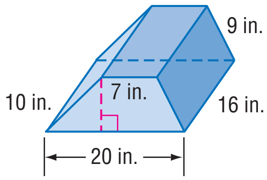
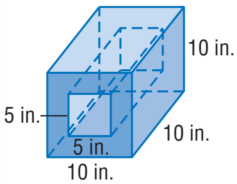
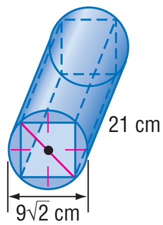
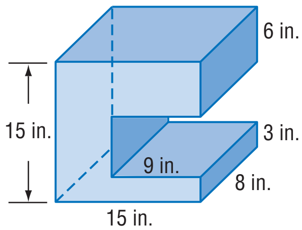


Find the volume of each solid to the nearest tenth.

4.


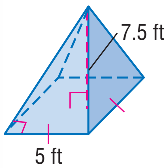
5.


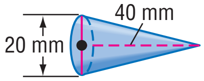
6.


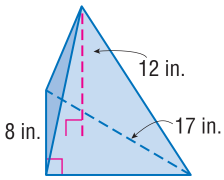
Lesson 13-2

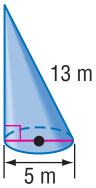
(pages 696–701)

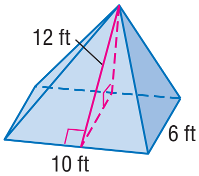
Find the volume of each cone or pyramid. Round to the nearest tenth if necessary.

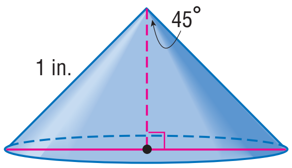
1.


2.


3.


4.


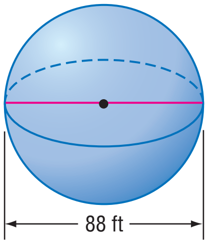
5.


6.


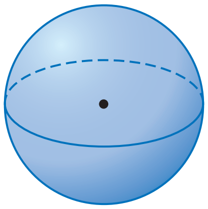
Lesson 13-3 (pages 702–706)

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

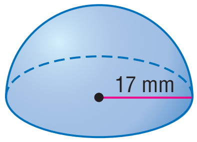
1.



2. $C = 4$ m



3.



4. The diameter of the sphere is 3 cm.

5. The radius of the hemisphere is $7\sqrt{2}$ m.

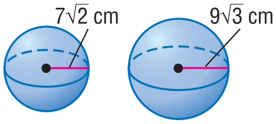
6. The diameter of the hemisphere is 90 ft.

7. The radius of the sphere is 0.5 in.

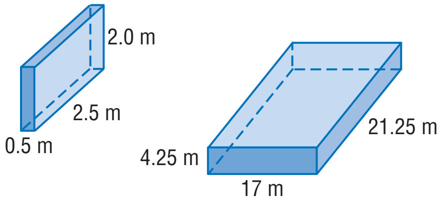
Lesson 13-4 (pages 707–713)

Determine whether each pair of solids are *similar, congruent,* or *neither.*

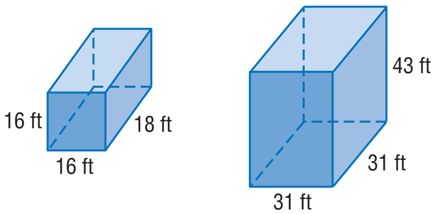
1.



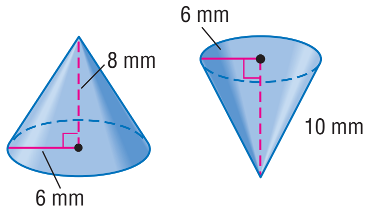
2.



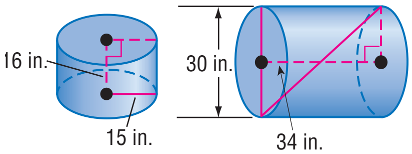
3.



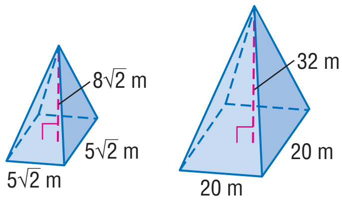
4.



5.



6.



Lesson 13-5 (pages 714–719)

Graph the rectangular solid that contains the given point and the origin. Label the coordinates of each vertex.

1. $A(3, -3, -3)$
2. $E(-1, 2, -3)$
3. $I(3, -1, 2)$
4. $Z(2, -1, 3)$
5. $Q(-4, -2, -4)$
6. $Y(-3, 1, -4)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint, *M*, of the segment joining the pair of points.

7. $A(-3, 3, 1)$ and $B(3, -3, -1)$
8. $O(2, -1, -3)$ and $P(-2, 4, -4)$
9. $D(0, -5, -3)$ and $E(0, 5, 3)$
10. $J(-1, 3, 5)$ and $K(3, -5, -3)$
11. $A(2, 1, 6)$ and $Z(-4, -5, -3)$
12. $S(-8, 3, -5)$ and $T(6, -1, 2)$

Mixed Problem Solving and Proof

Chapter 1 Points, Lines, Planes, and Angles

(pages 4–59)

ARCHITECTURE **For Exercises 1–4, use the following information.**
The Burj Al Arab in Dubai, United Arab Emirates, is one of the world's tallest hotels. *(Lesson 1-1)*

1. Trace the outline of the building on your paper.
2. Label three different planes suggested by the outline.
3. Highlight three lines in your drawing that, when extended, do not intersect.
4. Label three points on your sketch. Determine if they are coplanar and collinear.

SKYSCRAPERS **For Exercises 5–7, use the following information.** *(Lesson 1-2)*

Tallest Buildings in San Antonio, TX	
Name	**Height (ft)**
Tower of the Americas	622
Marriot Rivercenter	546
Weston Centre	444
Tower Life	404

Source: www.skyscrapers.com

5. What is the precision for the measures of the heights of the buildings?
6. What does the precision mean for the measure of the Tower of the Americas?
7. What is the difference in height between Weston Centre and Tower Life?

PERIMETER **For Exercises 8–11, use the following information.** *(Lesson 1-3)*
The coordinates of the vertices of $\triangle ABC$ are $A(0, 6)$, $B(-6, -2)$, and $C(8, -4)$. Round to the nearest tenth.

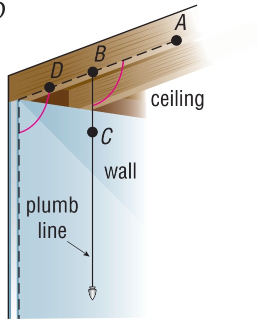
8. Find the perimeter of $\triangle ABC$.
9. Find the coordinates of the midpoints of each side of $\triangle ABC$.
10. Suppose the midpoints are connected to form a triangle. Find the perimeter of this triangle.
11. Compare the perimeters of the two triangles.

12. **TRANSPORTATION** Mile markers are used to name the exits on Interstate 70 in Kansas. The exit for Hays is 3 miles farther than halfway between Exits 128 and 184. What is the exit number for the Hays exit? *(Lesson 1-3)*

13. **ENTERTAINMENT** The Ferris wheel at the Navy Pier in Chicago has forty gondolas. What is the measure of an angle with a vertex that is the center of the wheel and with sides that are two consecutive spokes on the wheel? Assume that the gondolas are equally spaced. *(Lesson 1-4)*

CONSTRUCTION **For Exercises 14–15, use the following information.**
A framer is installing a cathedral ceiling in a newly built home. A protractor and a plumb bob are used to check the angle at the joint between the ceiling and wall. The wall is vertical, so the angle between the vertical plumb line and the ceiling is the same as the angle between the wall and the ceiling. *(Lesson 1-5)*

14. How are $\angle ABC$ and $\angle CBD$ related?
15. If $m\angle ABC = 110$, what is $m\angle CBD$?



STRUCTURES **For Exercises 16–17, use the following information.** *(Lesson 1-6)*
The picture shows the Hongkong and Shanghai Bank located in Hong Kong, China.

16. Name five different polygons suggested by the picture, classifying them by the number of sides.
17. Classify each polygon you identified as *convex* or *concave* and *regular* or *irregular*.

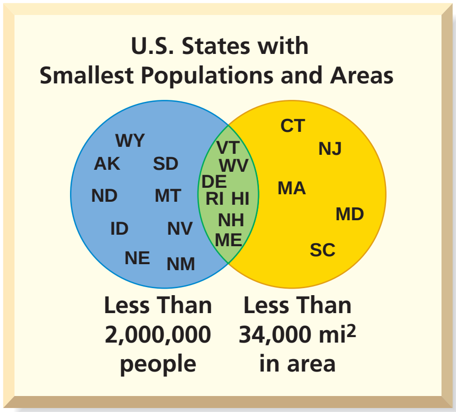
POPULATION **For Exercises 1–2, use the table showing the population density for various states in 1960, 1980, and 2000. The figures represent the number of people per square mile.** *(Lesson 2-1)*

State	1960	1980	2000
CA	100.4	151.4	217.2
CT	520.6	637.8	702.9
DE	225.2	307.6	401.0
HI	98.5	150.1	188.6
MI	137.7	162.6	175.0

Source: U.S. Census Bureau

1. Find a counterexample for the following statement. The population density for each state in the table increased by at least 30 during each 20-year period.
2. Write two conjectures for the year 2010.

STATES **For Exercises 3–5, refer to the Venn diagram.** *(Lesson 2-2)*



Source: *World Almanac*

3. How many states have less than 2,000,000 people?
4. How many states have less than 34,000 square miles in area?
5. How many states have less than 2,000,000 people and are less than 34,000 square miles in area?

LITERATURE **For Exercises 6–7, use the following quote from Lewis Carroll's *Alice's Adventures in Wonderland.*** *(Lesson 2-3)*

"Then you should say what you mean," the March Hare went on.
"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."
"Not the same thing a bit!" said the Hatter.

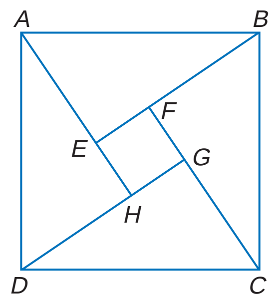
6. Who is correct? Explain.
7. How are the phrases *say what you mean* and *mean what you say* related?

8. **AIRLINE SAFETY** Airports in the United States post a sign stating *If any unknown person attempts to give you any items including luggage to transport on your flight, do not accept it and notify airline personnel immediately.* Write a valid conclusion to the hypothesis, *If a person Candace does not know attempts to give her an item to take on her flight, . . .* *(Lesson 2-4)*

9. **PROOF** Write a paragraph proof to show that $\overline{AB} \cong \overline{CD}$ if B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. *(Lesson 2–5)*

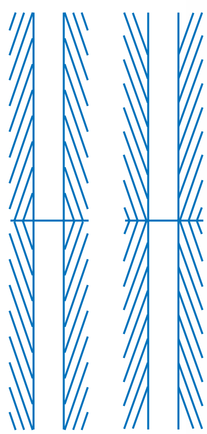
10. **CONSTRUCTION** Engineers consider the expansion and contraction of materials used in construction. The coefficient of linear expansion, k, is dependent on the change in length and the change in temperature and is found by the formula, $k = \frac{\Delta \ell}{\ell(T - t)}$. Solve this formula for T and justify each step. *(Lesson 2-6)*

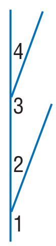
11. **PROOF** Write a two-column proof. *(Lesson 2-7)*
Given: $ABCD$ has 4 congruent sides.
$DH = BF = AE$; $EH = FE$
Prove: $AB + BE + AE = AD + AH + DH$



ILLUSIONS **This drawing was created by German psychologist Wilhelm Wundt.** *(Lesson 2-8)*

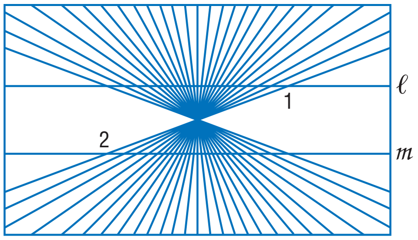
12. Describe the relationship between each pair of vertical lines.
13. A close-up of the angular lines is shown below. If $\angle 4 \cong \angle 2$, write a two-column proof to show that $\angle 3 \cong \angle 1$.





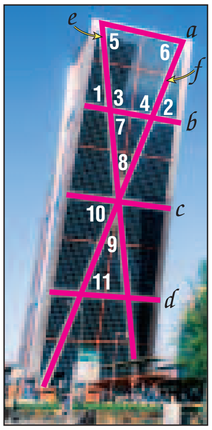
CONTENTS

1. **OPTICAL ILLUSIONS** Lines ℓ and m are parallel, but appear to be bowed due to the transversals drawn through ℓ and m. Make a conjecture about the relationship between $\angle 1$ and $\angle 2$. *(Lesson 3-1)*



ARCHITECTURE For Exercises 2–10, use the following information. *(Lesson 3-2)*

The picture shows one of two towers of the Puerta de Europa in Madrid, Spain. Lines a, b, c, and d are parallel. The lines are cut by transversals e and f. If $m\angle 1 = m\angle 2 = 75$, find the measure of each angle.

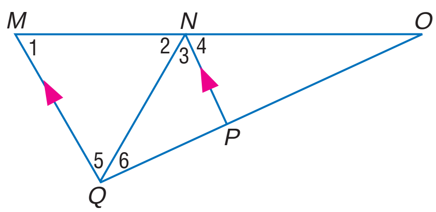


2. $\angle 3$
3. $\angle 4$
4. $\angle 5$
5. $\angle 6$
6. $\angle 7$
7. $\angle 8$
8. $\angle 9$
9. $\angle 10$
10. $\angle 11$

11. **PROOF** Write a two-column proof. *(Lesson 3-2)*

Given: $\overline{MQ} \parallel \overline{NP}$
$\angle 4 \cong \angle 3$

Prove: $\angle 1 \cong \angle 5$



12. **EDUCATION** Between 1995 and 2000, the average cost for tuition and fees for American universities increased by an average rate of \$84.20 per year. In 2000, the average cost was \$2600. If costs increase at the same rate, what will the total average cost be in 2010? *(Lesson 3-3)*

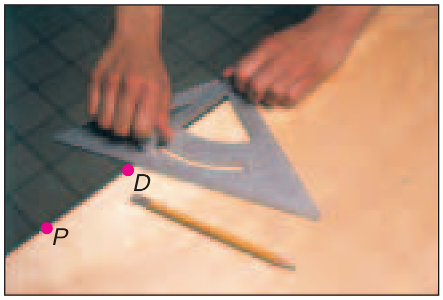
RECREATION For Exercises 13 and 14, use the following information. *(Lesson 3-4)*

The Three Forks community swimming pool holds 74,800 gallons of water. At the end of the summer, the pool is drained and winterized.

13. If the pool drains at the rate of 1200 gallons per hour, write an equation to describe the number of gallons left after x hours.

14. How many hours will it take to drain the pool?

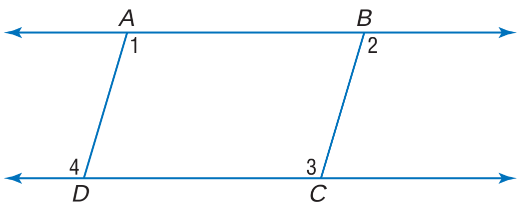
15. **CONSTRUCTION** An *engineer and carpenter square* is used to draw parallel line segments. Martin makes two cuts at an angle of 120° with the edge of the wood through points D and P. Explain why these cuts will be parallel. *(Lesson 3-5)*



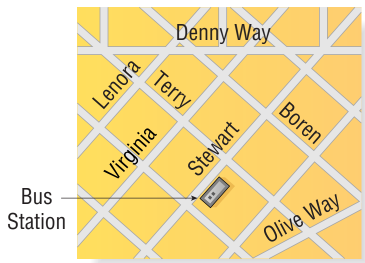
16. **PROOF** Write a two-column proof. *(Lesson 3-5)*

Given: $\angle 1 \cong \angle 3$
$\overline{AB} \parallel \overline{DC}$

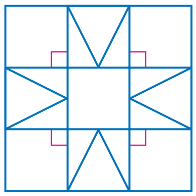
Prove: $\overline{BC} \parallel \overline{AD}$



17. **CITIES** The map shows a portion of Seattle, Washington. Describe a segment that represents the shortest distance from the Bus Station to Denny Way. Can you walk the route indicated by your segment? Explain. *(Lesson 3-6)*

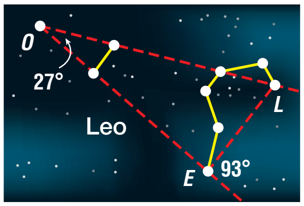


QUILTING **For Exercises 1 and 2, trace the quilt pattern square below.** *(Lesson 4-1)*

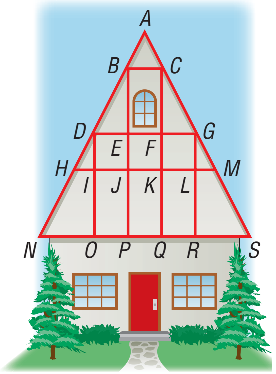


1. Shade all right triangles red. Do these triangles appear to be scalene or isosceles? Explain.
2. Shade all acute triangles blue. Do these triangles appear to be scalene, isoscles, or equilateral? Explain.

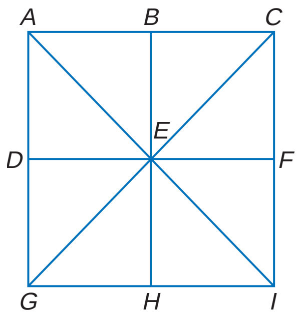
3. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form $\triangle LEO$. If the angles have measures as shown in the figure, find $m\angle OLE$. *(Lesson 4-2)*



4. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent. *(Lesson 4-3)*

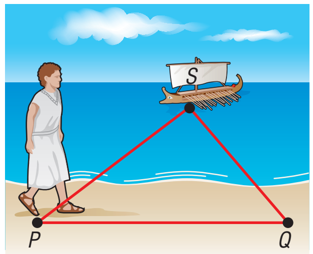


RECREATION **For Exercises 5–7, use the following information.**
Tapatan is a game played in the Philippines on a square board, like the one shown at the top right. Players take turns placing each of their three pieces on a different point of intersection. After all the pieces have been played, the players take turns moving a piece along a line to another intersection. A piece cannot jump over another piece. A player who gets all their pieces in a straight line wins. Point E bisects all four line segments that pass through it. All sides are congruent, and the diagonals are congruent. Suppose a letter is assigned to each intersection. *(Lesson 4-4)*



5. Is $\triangle GHE \cong \triangle CBE$? Explain.
6. Is $\triangle AEG \cong \triangle IEG$? Explain.
7. Write a flow proof to show that $\triangle ACI \cong \triangle CAG$.

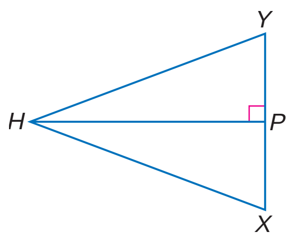
8. **HISTORY** It is said that Thales determined the distance from the shore to the Greek ships by sighting the angle to the ship from a point P on the shore, walking to point Q, and then sighting the angle to the ship from Q. He then reproduced the angles on the other side of $\overline{PQ}$ and continued these lines until they intersected. Is this method valid? Explain. *(Lesson 4-5)*



9. **PROOF** Write a two-column proof. *(Lesson 4-6)*

Given: $\overline{PH}$ bisects $\angle YHX$.
$\overline{PH} \perp \overline{YX}$

Prove: $\triangle YHX$ is an isosceles triangle.

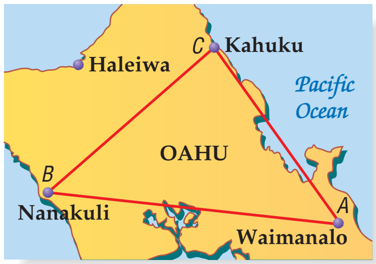


10. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$. *(Lesson 4-7)*

CONSTRUCTION **For Exercises 1–4, draw a large, acute scalene triangle. Use a compass and straightedge to make the required constructions.** *(Lesson 5-1)*

1. Find the circumcenter. Label it C.
2. Find the centroid of the triangle. Label it D.
3. Find the orthocenter. Label it O.
4. Find the incenter of the triangle. Label it I.

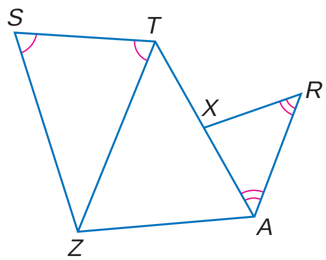
RECREATION **For Exercises 5–7, use the following information.** *(Lesson 5-2)*
Kailey plans to fly over the route marked on the map of Oahu in Hawaii.



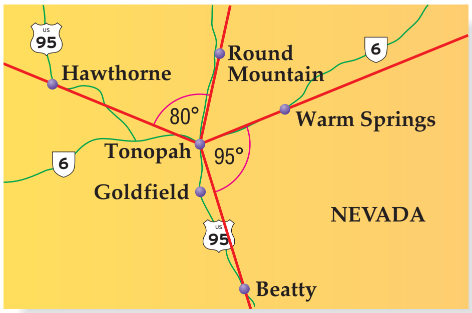
5. The measure of angle A is two degrees more than the measure of angle B. The measure of angle C is fourteen degrees less than twice the measure of angle B. What are the measures of the three angles?
6. Write the lengths of the legs of Kailey's trip in order from least to greatest.
7. The length of the entire trip is about 70 miles. The middle leg is 12.5 miles greater than one-half the length of the shortest leg. The longest leg is 12.5 miles greater than three-fourths of the shortest leg. What are the lengths of the legs of the trip?
8. **LAW** A man is accused of comitting a crime. If the man is telling the truth when he says, "I work every Tuesday from 3:00 P.M. to 11:00 P.M.," what fact about the crime could be used to prove by indirect reasoning that the man was innocent? *(Lesson 5-3)*

TRAVEL **For Exercises 9 and 10, use the following information.**
The total air distance to fly from Bozeman, Montana, to Salt Lake City, Utah, to Boise, Idaho is just over 634 miles.

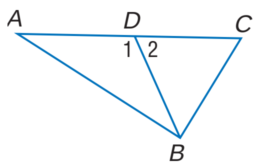
9. Write an indirect proof to show that at least one of the legs of the trip is longer than 317 miles. *(Lesson 5-3)*
10. The air distance from Bozeman to Salt Lake City is 341 miles and the distance from Salt Lake to Boise is 294 miles. Find the range for the distance from Bozeman to Boise. *(Lesson 5-4)*
11. **PROOF** Write a two-column proof.
Given: $\angle ZST \cong \angle ZTS$
$\angle XRA \cong \angle XAR$
$TA = 2AX$
Prove: $2XR + AZ > SZ$
(Lesson 5-4)



12. **GEOGRAPHY** The map shows a portion of Nevada. The distance from Tonopah to Round Mountain is the same as the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Use the angle measures to determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty. *(Lesson 5-5)*



13. **PROOF** Write a two-column proof. *(Lesson 5-5)*
Given: $\overline{DB}$ is a median of $\triangle ABC$.
$m\angle 1 > m\angle 2$
Prove: $m\angle C > m\angle A$



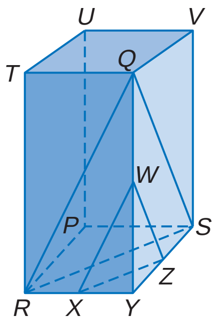
1. **TOYS** In 2000, \$34,554,900,000 was spent on toys in the U.S. The U.S. population in 2000 was 281,421,906, with 21.4% of the population 14 years and under. If all of the toys purchased in 2000 were for children 14 years and under, what was the average amount spent per child? *(Lesson 6-1)*

QUILTING For Exercises 2–4, use the following information. *(Lesson 6-2)*
Felicia found a pattern for a quilt square. The pattern measures three-quarters of an inch on a side. Felicia wants to make a quilt that is 77 inches by 110 inches when finished.

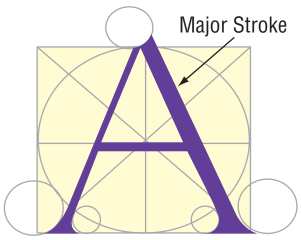
2. If Felicia wants to use only whole quilt squares, what is the greatest side length she can use for each square?
3. How many quilt squares will she need for the quilt?
4. By what scale factor will she need to increase the pattern for the quilt square?

PROOF For Exercises 5 and 6, write a paragraph proof. *(Lesson 6-3)*

5. **Given:** $\triangle WYX \sim \triangle QYR$, $\triangle ZYX \sim \triangle SYR$
   Prove: $\triangle WYZ \sim \triangle QYS$
6. **Given:** $\overline{WX} \parallel \overline{QR}$, $\overline{ZX} \parallel \overline{SR}$
   Prove: $\overline{WZ} \parallel \overline{QS}$

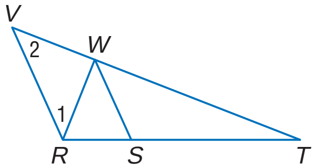


HISTORY For Exercises 7 and 8, use the following information. *(Lesson 6-4)*
In the fifteenth century, mathematicians and artists tried to construct the perfect letter. Damiano da Moile used a square as a frame to design the letter "A" as shown in the diagram. The thickness of the major stroke of the letter was to be $\frac{1}{12}$ of the height of the letter.

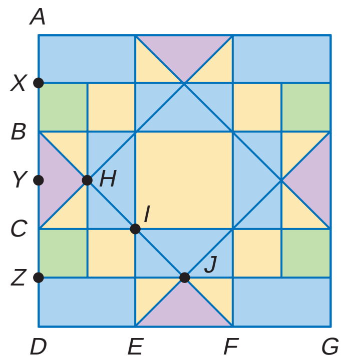


7. Explain why the bar through the middle of the A is half the length between the outside bottom corners of the sides of the letter.
8. If the letter were 3 centimeters tall, how wide would the major stroke of the A be?

9. **PROOF** Write a two-column proof. *(Lesson 6-5)*
   Given: $\overline{WS}$ bisects $\angle RWT$. $\angle 1 \cong \angle 2$
   Prove: $\frac{VW}{WT} = \frac{RS}{ST}$

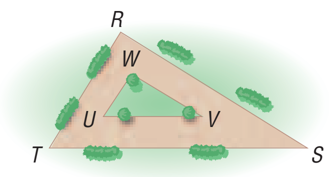


ART For Exercises 10 and 11, use the diagram of a square mosaic tile. $AB = BC = CD = \frac{1}{3}AD$ and $DE = EF = FG = \frac{1}{3}DG$. *(Lesson 6-5)*



10. What is the ratio of the perimeter of $\triangle BDF$ to the perimeter of $\triangle BCI$? Explain.
11. Find two triangles such that the ratio of their perimeters is 2 : 3. Explain.

12. **TRACK** A triangular track is laid out as shown. $\triangle RST \sim \triangle WVU$. If $UV = 500$ feet, $VW = 400$ feet, $UW = 300$ feet, and $ST = 1000$ feet, find the perimeter of $\triangle RST$. *(Lesson 6-5)*



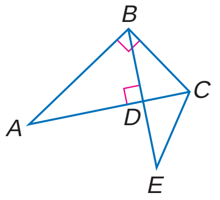
13. **BANKING** Ashante has \$5000 in a savings account with a yearly interest rate of 2.5%. The interest is compounded twice per year. What will be the amount in the savings account after 5 years? *(Lesson 6-6)*

Mixed Problem Solving and Proof

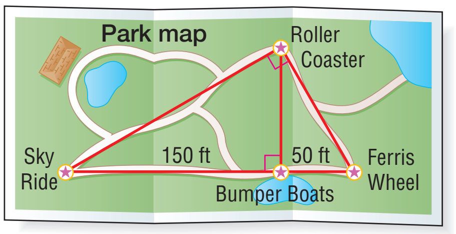
1. **PROOF** Write a two-column proof. *(Lesson 7-1)*

Given: D is the midpoint of $\overline{BE}$, $\overline{BD}$ is an altitude of right triangle $\triangle ABC$

Prove: $\frac{AD}{DE} = \frac{DE}{DC}$

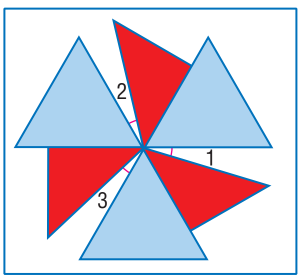


2. **AMUSEMENT PARKS** The map shows the locations of four rides at an amusement park. Find the length of the path from the roller coaster to the bumper boats. Round to the nearest tenth. *(Lesson 7-1)*



3. **CONSTRUCTION** Carlotta drew a diagram of a right triangular brace with side measures of 2.7 centimeters, 3.0 centimeters, and 5.3 centimeters. Is the diagram correct? Explain. *(Lesson 7-2)*

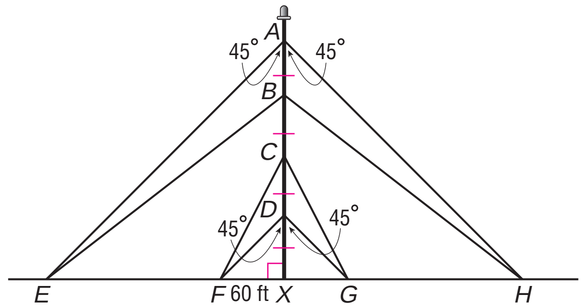
DESIGN For Exercises 4–5, use the following information. *(Lesson 7-3)*
Kwan designed the pinwheel. The blue triangles are congruent equilateral triangles each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of a blue triangle.



4. If angles 1, 2, and 3 are congruent, find the measure of each angle.

5. Find the perimeter of the pinwheel. Round to the nearest inch.

COMMUNICATION For Exercises 6–9, use the following information. *(Lesson 7-4)*
The diagram shows a radio tower secured by four pairs of guy wires that are equally spaced apart with $DX = 60$ feet. Round to the nearest tenth if necessary.



6. Name the isosceles triangles in the diagram.

7. Find $m\angle BEX$ and $m\angle CFX$.

8. Find AE, EB, CF, and DF.

9. Find the total amount of wire used to support the tower.

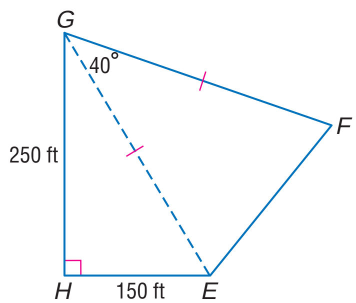
10. **METEOROLOGY** A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is 47°, how high is the cloud ceiling? *(Lesson 7-5)*

GARDENING For Exercises 11 and 12, use the information below. *(Lesson 7-6)*
A flower bed at Magic City Rose Garden is in the shape of an obtuse scalene triangle with the shortest side measuring 7.5 feet. Another side measures 14 feet and the measure of the opposite angle is 103°.

11. Find the measures of the other angles of the triangle. Round to the nearest degree.

12. Find the perimeter of the garden. Round to the nearest tenth.

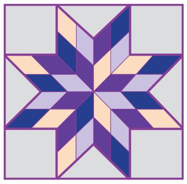
13. **HOUSING** Mr. and Mrs. Abbott bought a lot at the end of a cul-de-sac. They want to build a fence on three sides of the lot, excluding $\overline{HE}$. To the nearest foot, how much fencing will they need to buy? *(Lesson 7-7)*



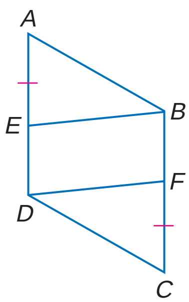
ENGINEERING **For Exercises 1–2, use the following information.**
The London Eye in London, England, is the world's largest observation wheel. The ride has 32 enclosed capsules for riders. *(Lesson 8-1)*

1. Suppose each capsule is connected with a straight piece of metal forming a 32-gon. Find the sum of the measures of the interior angles.
2. What is the measure of one interior angle of the 32-gon?

3. **QUILTING** The quilt square shown is called the Lone Star pattern. Describe two ways that the quilter could ensure that the pieces will fit properly. *(Lesson 8-2)*

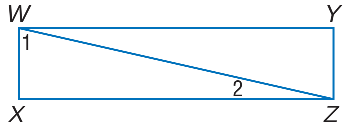


4. **PROOF** Write a two-column proof. *(Lesson 8-3)*
Given: $\square ABCD$, $\overline{AE} \cong \overline{CF}$
Prove: Quadrilateral *EBFD* is a parallelogram.

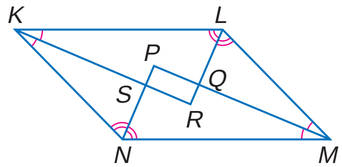


5. **MUSIC** Why will the keyboard stand shown always remain parallel to the floor? *(Lesson 8-3)*

6. **PROOF** Write a two-column proof. *(Lesson 8-4)*
Given: $\square WXZY$, $\angle 1$ and $\angle 2$ are complementary.
Prove: *WXZY* is a rectangle.

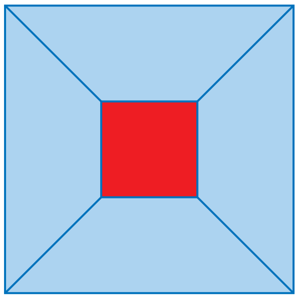


7. **PROOF** Write a paragraph proof. *(Lesson 8-4)*
Given: $\square KLMN$
Prove: *PQRS* is a rectangle.



8. **CONSTRUCTION** Mr. Redwing is building a sandbox. He placed stakes at what he believes will be the four vertices of a square with a distance of 5 feet between each stake. How can he be sure that the sandbox will be a square? *(Lesson 8-5)*

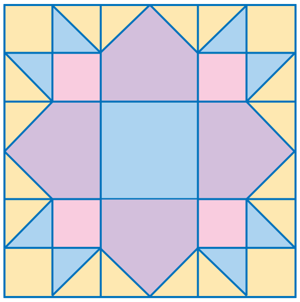
DESIGN **For Exercises 9 and 10, use the square floor tile design shown below.** *(Lesson 8-6)*



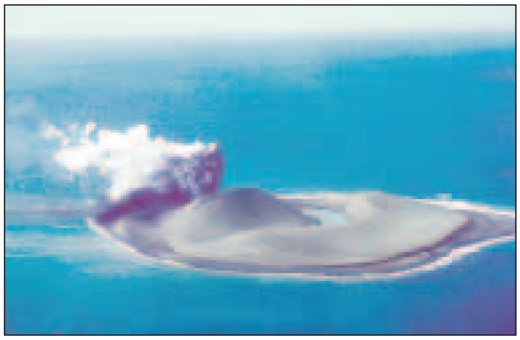
9. Explain how you know that the trapezoids in the design are isosceles.
10. The perimeter of the floor tile is 48 inches, and the perimeter of the interior red square is 16 inches. Find the perimeter of one trapezoid.

11. **PROOF** Position a quadrilateral on the coordinate plane with vertices $Q(-a, 0)$, $R(a, 0)$, $S(b, c)$, and $T(-b, c)$. Prove that the quadrilateral is an isosceles trapezoid. *(Lesson 8-7)*

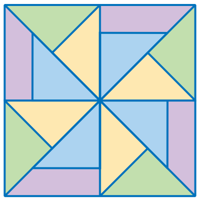
QUILTING **For Exercises 1 and 2, use the diagram of a quilt square.** *(Lesson 9-1)*



1. How many lines of symmetry are there for the entire quilt square?
2. Consider different sections of the quilt square. Describe at least three different lines of reflection and the figures reflected in those lines.
3. **ENVIRONMENT** A cloud of dense gas and dust pours out of Surtsey, a volcanic island off the south coast of Iceland. If the cloud blows 40 miles north and then 30 miles east, make a sketch to show the translation of the smoke particles. Then find the distance of the shortest path that would take the particles to the same position. *(Lesson 9-2)*



ART **For Exercises 4–7, use the mosaic tile.**

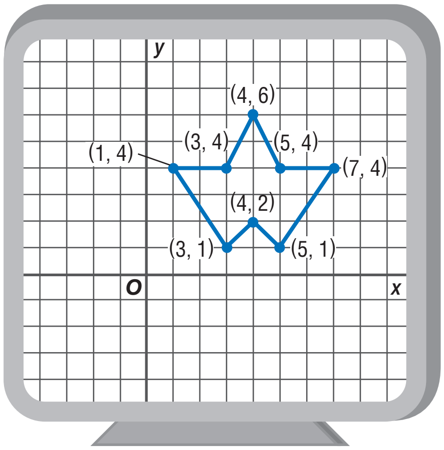


4. Identify the order and magnitude of rotation that takes a yellow triangle to a blue triangle. *(Lesson 9-3)*
5. Identify the order and magnitude of rotation that takes a blue triangle to a yellow triangle. *(Lesson 9-3)*
6. Identify the magnitude of rotation that takes a trapezoid to a consecutive trapezoid. *(Lesson 9-3)*
7. Is the mosaic design a tessellation? Explain. *(Lesson 9-4)*
8. **CRAFTS** Eduardo found a pattern for cross-stitch on the Internet. The pattern measures 2 inches by 3 inches. He would like to enlarge the piece to 4 inches by 6 inches. The copy machine available to him enlarges 150% or less by increments of whole number percents. Find two whole number percents by which he can consecutively enlarge the piece and get as close to the desired dimensions as possible without exceeding them. *(Lesson 9-5)*

AVIATION **For Exercises 9 and 10, use the following information.** *(Lesson 9-6)*
A small aircraft flies due south at an average speed of 190 miles per hour. The wind is blowing due west at 30 miles per hour.

9. Draw a diagram using vectors to represent this situation.
10. Find the resultant velocity and direction of the plane.

GRAPHICS **For Exercises 11–14, use the graphic shown on the computer screen.** *(Lesson 9-7)*



11. Suppose you want the figure to move to Quadrant III but be upside down. Write two matrices that make this transformation, if they are applied consecutively.
12. Write one matrix that can be used to do the same transformation as in Exercise 11. What type of transformation is this?
13. Compare the two matrices in Exercise 11 to the matrix in Exercise 12. What do you notice?
14. Write the vertex matrix for the figure in Quadrant III and graph it on the coordinate plane.

1. **CYCLING** A bicycle tire travels about 50.27 inches during one rotation of the wheel. What is the diameter of the tire? *(Lesson 10-1)*

SPACE For Exercises 2–4, use the following information. *(Lesson 10-2)*
School children were recently surveyed about what they believe to be the most important reason to explore Mars. They were given five choices and the table below shows the results.

Reason to Visit Mars	Number of Students
Learn about life beyond Earth	910
Learn more about Earth	234
Seek potential for human inhabitance	624
Use as a base for further exploration	364
Increase human knowledge	468

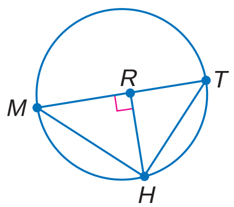
Source: *USA TODAY*

2. If you were to construct a circle graph of this data, how many degrees would be allotted to each category?
3. Describe the type of arc associated with each category.
4. Construct a circle graph for these data.

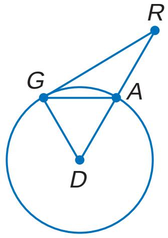
5. **CRAFTS** Yvonne uses wooden spheres to make paperweights to sell at craft shows. She cuts off a flat surface for each base. If the original sphere has a radius of 4 centimeters and the diameter of the flat surface is 6 centimeters, what is the height of the paperweight? *(Lesson 10-3)*

6. **PROOF** Write a two-column proof for the following information. *(Lesson 10-4)*

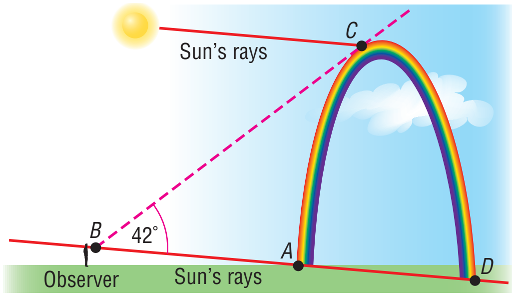
Given: $\overset{\frown}{MHT}$ is a semicircle.
$\overline{RH} \perp \overline{TM}$
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



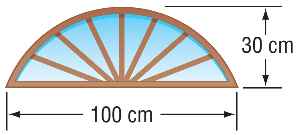
7. **PROOF** Write a paragraph proof. *(Lesson 10-5)*
Given: $\overline{GR}$ is tangent to $\odot D$ at G.
$\overline{AG} \cong \overline{DG}$
Prove: $\overline{AG}$ bisects $\overline{RD}$.



8. **METEOROLOGY** A rainbow is really a full circle with a center at a point in the sky directly opposite the Sun. The position of a rainbow varies according to the viewer's position, but its angular size, $\angle ABC$, is always 42°. If $m\overset{\frown}{CD} = 160$, find the measure of the visible part of the rainbow, $m\overset{\frown}{AC}$. *(Lesson 10-6)*

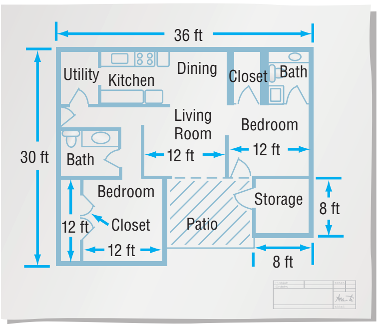


9. **CONSTRUCTION** An arch over an entrance is 100 centimeters wide and 30 centimeters high. Find the radius of the circle that contains the arch. *(Lesson 10-7)*



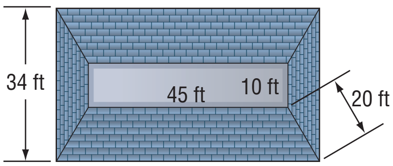
10. **SPACE** Objects that have been left behind in Earth's orbit from space missions are called "space junk." These objects are a hazard to current space missions and satellites. Eighty percent of space junk orbits Earth at a distance of 1,200 miles from the surface of Earth, which has a diameter of 7,926 miles. Write an equation to model the orbit of 80% of space junk with Earth's center at the origin. *(Lesson 10-8)*

REMODELING For Exercises 1–3, use the following information.
The diagram shows the floor plan of the home that the Summers are buying. They want to replace the patio with a larger sunroom to increase their living space by one-third. *(Lesson 11-1)*



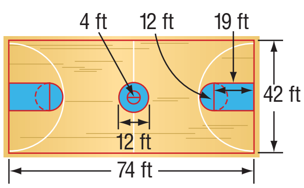
1. Excluding the patio and storage area, how many square feet of living area are in the current house?
2. What area should be added to the house to increase the living area by one-third?
3. The Summers want to connect the bedroom and storage area with the sunroom. What will be the dimensions of the sunroom?

HOME REPAIR For Exercises 4 and 5, use the following information.
Scott needs to replace the shingles on the roof of his house. The roof is composed of two large isosceles trapezoids, two smaller isosceles trapezoids, and a rectangle. Each trapezoid has the same height. *(Lesson 11-2)*

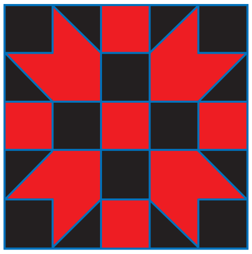


4. Find the height of the trapezoids.
5. Find the area of the roof to be covered by shingles.

6. **SPORTS** The Moore High School basketball team wants to paint their basketball court as shown. They want the center circle and the free throw areas painted blue. What is the area of the court that they will paint blue? *(Lesson 11-3)*

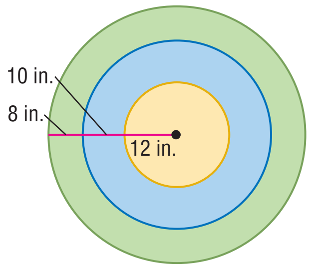


MUSEUMS For Exercises 7–9, use the following information.
The Hyalite Hills Museum plans to install the square mosaic pattern shown below in the entry hall. It is 10 feet on each side with each small black or red square tile measuring 2 feet on each side. *(Lesson 11-4)*

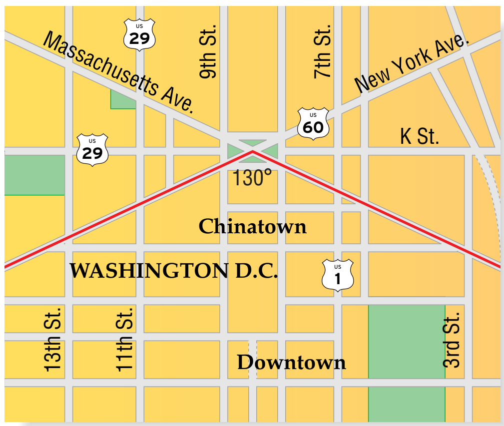


7. Find the area of black tiles.
8. Find the area of red tiles.
9. Which is greater, the total perimeter of the red tiles or the total perimeter of the black tiles? Explain.

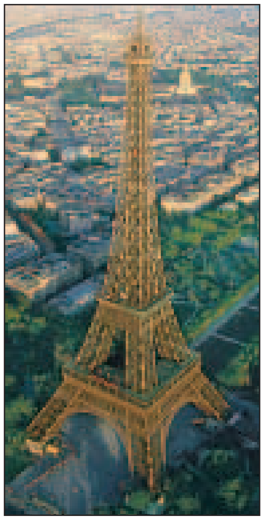
10. **GAMES** If the dart lands on the target, find the probability that it lands in the blue region. *(Lesson 11-5)*



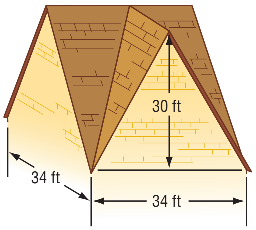
11. **ACCOMMODATIONS** The convention center in Washington D.C., lies in the northwest sector of the city between New York and Massachusetts Avenues, which intersect at a 130° angle. If the amount of hotel space is evenly distributed over an area with that intersection as the center and a radius of 1.5 miles, what is the probability that a vistor, randomly assigned to a hotel, will be housed in the sector containing the convention center? *(Lesson 11-5)*



1. **ARCHITECTURE** Sketch an orthogonal drawing of the Eiffel Tower. *(Lesson 12-1)*



2. **CONSTRUCTION** The roof shown below is a hip-and-valley style. Use the dimensions given to find the area of the roof that would need to be shingled. *(Lesson 12-2)*



3. **AERONAUTICAL ENGINEERING** The surface area of the wing on an aircraft is used to determine a design factor known as wing loading. If the total weight of the aircraft and its load is w and the total surface area of its wings is s, then the formula for the wing loading factor, ℓ, is $\ell = \frac{w}{s}$. If the wing loading factor is exceeded, the pilot must either reduce the fuel load or remove passengers or cargo. Find the wing loading factor for a plane if it had a take-off weight of 750 pounds and the surface area of the wings was 532 square feet. *(Lesson 12-2)*

4. **MANUFACTURING** Many baking pans are given a special nonstick coating. A rectangular cake pan is 9 inches by 13 inches by 2 inches deep. What is the area of the inside of the pan that needs to be coated? *(Lesson 12-3)*

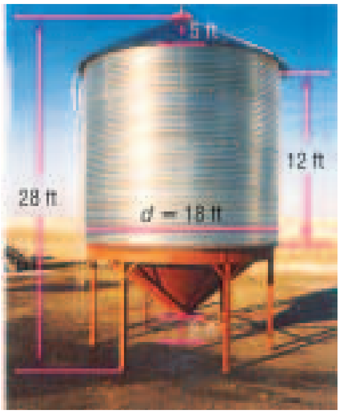
5. **COMMUNICATIONS** Coaxial cable is used to transmit long-distance telephone calls, cable television programming, and other communications. A typical coaxial cable contains 22 copper tubes and has a diameter of 3 inches. What is the lateral area of a coaxial cable that is 500 feet long? *(Lesson 12-4)*

COLLECTIONS **For Exercises 6 and 7, use the following information.**
Soledad collects unique salt-and-pepper shakers. She inherited a pair of tetrahedral shakers from her mother. *(Lesson 12-5)*

6. Each edge of a shaker measures 3 centimeters. Make a sketch of one shaker.

7. Find the total surface area of one shaker.

8. **FARMING** The picture below shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions shown in the diagram to find the entire surface area of the bin with a conical top and bottom. Write the exact answer and the answer rounded to the nearest square foot. *(Lesson 12-6)*



GEOGRAPHY **For Exercises 9–11, use the following information.**
Joaquin is buying Dennis a globe for his birthday. The globe has a diameter of 16 inches. *(Lesson 12-7)*

9. What is the surface area of the globe?

10. If the diameter of Earth is 7926 miles, find the surface area of Earth.

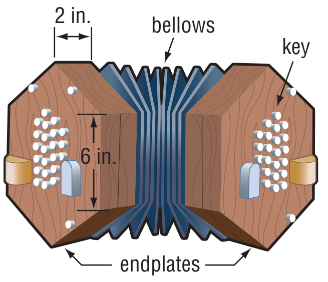
11. The continent of Africa occupies about 11,700,000 square miles. How many square inches will be used to represent Africa on the globe?

(t)Yann Arthus-Bertrand/CORBIS, (c)courtesy M-K Distributors, Conrad MT, (b)Aaron Haupt

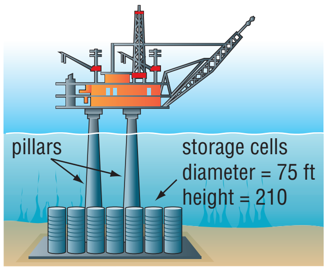
1. **METEOROLOGY** The TIROS weather satellites were a series of weather satellites, the first being launched on April 1, 1960. These satellites carried television and infrared cameras and were covered by solar cells. If the cylinder-shaped body of a TIROS had a diameter of 42 inches and a height of 19 inches, what was the volume available for carrying instruments and cameras? Round to the nearest tenth. *(Lesson 13-1)*

2. **SPACECRAFT** The smallest manned spacecraft, used by astronauts for jobs outside the Space Shuttle, is the Manned Maneuvering Unit. It is 4 feet tall, 2 feet 8 inches wide, and 3 feet 8 inches deep. Find the volume of this spacecraft in cubic feet. Round to the nearest tenth. *(Lesson 13-1)*

3. **MUSIC** To play a concertina, you push and pull the end plates and press the keys. The air pressure causes vibrations of the metal reeds that make the notes. When fully expanded, the concertina is 36 inches from end to end. If the concertina is compressed, it is 7 inches from end to end. Find the volume of air in the instrument when it is fully expanded and when it is compressed. (*Hint:* There is no air in the endplates.) *(Lesson 13-1)*

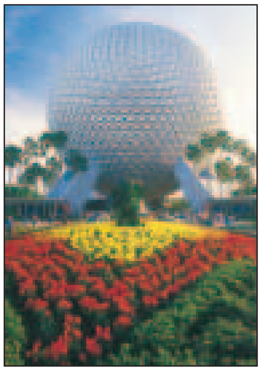


4. **ENGINEERING** The base of an oil drilling platform is made up of 24 concrete cylindrical cells. Twenty of the cells are used for oil storage. The pillars that support the platform deck rest on the four other cells. Find the total volume of the storage cells. *(Lesson 13-1)*



5. **HOME BUSINESS** Jodi has a home-based business selling homemade candies. She is designing a pyramid-shaped box for the candy. The base is a square measuring 14.5 centimeters on a side. The slant height of the pyramid is 16 centimeters. Find the volume of the box. Round to the nearest cubic centimeter. *(Lesson 13-2)*

ENTERTAINMENT For Exercises 6–10, use the following information.
Some people think that the Spaceship Earth geosphere at Epcot® in Disney World resembles a golf ball. The building is a sphere measuring 165 feet in diameter. A typical golf ball has a diameter of approximately 1.5 inches.



6. Find the volume of Spaceship Earth. Round to the nearest cubic foot. *(Lesson 13-3)*
7. Find the volume of a golf ball. Round to the nearest tenth. *(Lesson 13-3)*
8. What is the scale factor that compares Spaceship Earth to a golf ball? *(Lesson 13-4)*
9. What is the ratio of the volume of Spaceship Earth to the volume of a golf ball? *(Lesson 13-4)*
10. Suppose a six-foot-tall golfer plays golf with a 1.5 inch diameter golf ball. If the ratio between golfer and ball remains the same, how tall would a golfer need to be to use Spaceship Earth as a golf ball? *(Lesson 13-4)*

ASTRONOMY For Exercises 11 and 12, use the following information.
A museum has set aside a children's room containing objects suspended from the ceiling to resemble planets and stars. Suppose an imaginary coordinate system is placed in the room with the center of the room at (0, 0, 0). Three particular stars are located at $S(-10, 5, 3)$, $T(3, -8, -1)$, and $R(-7, -4, -2)$, where the coordinates represent the distance in feet from the center of the room. *(Lesson 13-5)*

11. Find the distance between each pair of stars.
12. Which star is farthest from the center of the room?